CSE 311: Foundations of Computing I

Induction Practice Day

1. An Odd Sum

Difficulty Level: Easier

Prove by induction that the sum of the first n odd positive integers is n^2.
Hint: You may find it helpful to write the sum of the first n odd positive integers as $1 + 3 + 5 + ... + (2n - 1)$.

2. Darn Divisibility

Difficulty Level: Medium

Prove by induction that for all $n \in \mathbb{N}$, $n^3 - n$ is divisible by 3.

3. Geometric Sum

Difficulty Level: Medium

Suppose that a and r are real numbers with $r \neq 1$. Prove by induction that for all $n \in \mathbb{N}$:

$$a + ar + ar^2 + ... + ar^n = \frac{a \cdot r^{n+1} - a}{r - 1}$$

4. Inequalities

Difficulty Level: Medium

Prove by induction that for all integers $n \geq 1$, we have:

$$\frac{1}{2n} \leq \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}$$

5. Sets of Integers

Difficulty Level: Hard

Prove by induction that for all integers $n \geq 1$, given a set of $n+1$ distinct positive integers, none exceeding $2n$, there is at least one integer in the set that divides another integer in the set.

6. Chessboard

Difficulty Level: Hard

A knight on a chessboard can move one space horizontally (in either direction) and two spaces vertically (in either direction), OR two spaces horizontally (in either direction) and one space vertically (in either direction). Suppose we have an infinite chessboard that starts at the bottom left corner $(0, 0)$, and extends upwards and to the right infinitely. The board is made up of all squares (m, n) where m, n are natural numbers that denote the column number and row number of the square, respectively. Use mathematical induction to show that a knight starting at $(0, 0)$ can visit every square using a finite sequence of moves.
Hint: Use induction on the variable $s = m + n$.