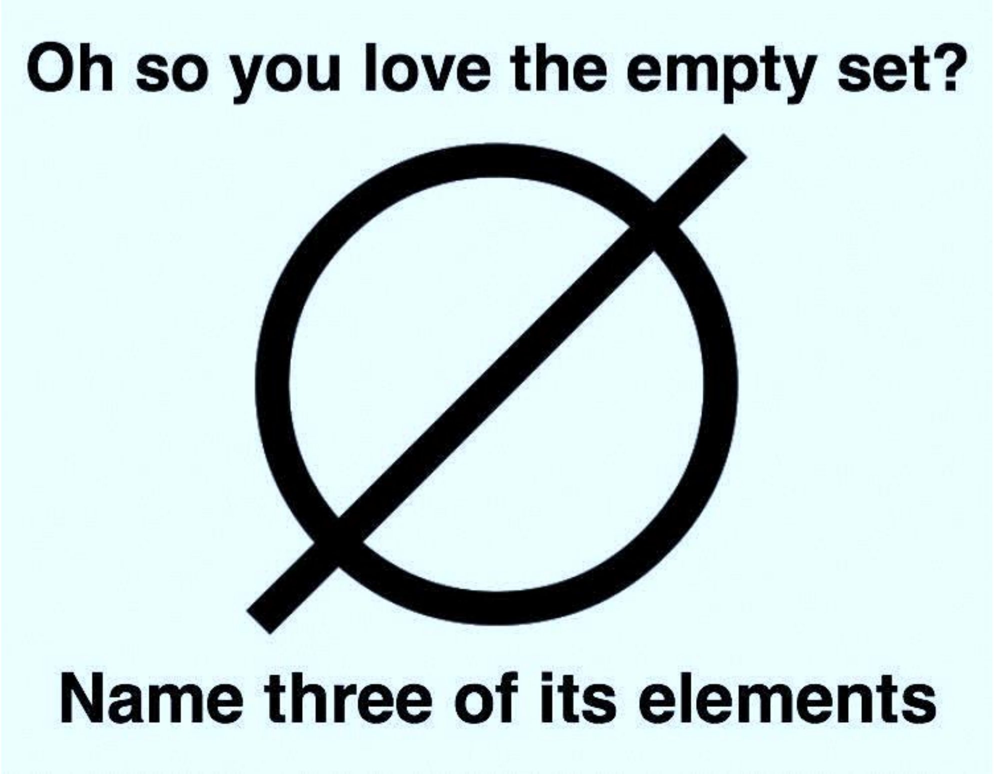


Set Theory

CSE 311: Foundations of Computing I
Lecture 11

Announcements

- HW3 late deadline is today at 11:59 pm
- HW4 has been posted, due 11:59 pm on Wednesday

Wrapping Number Theory

Recall

Let a, b, c, d and $m > 0$ be integers.

- If $a \equiv_m b$, then $b \equiv_m a$.
- If $a \equiv_m b$ and $c \equiv_m d$, then $a + c \equiv_m b + d$.
- If $a \equiv_m b$ and $c \equiv_m d$, then $ac \equiv_m bd$.
- If $a \equiv_m b$ and $b \equiv_m c$, then $a \equiv_m c$.
- $a \equiv_m b$ if and only if $a \,\%\, m = b \,\%\, m$.

Solving in Modular Arithmetic

Solve: $5 + x \equiv_{10} 9(32 - 2)$

Solving in Modular Arithmetic

Solve: $7x \equiv_{10} 1$

Solution: $x \equiv_{10} 3$ (Guess and check)

None of our properties so far help us solve this.

There is an algorithm to solve this called the Extended Euclidean Algorithm, if you're interested.

3 is called the multiplicative inverse of 7 modulo 10, i.e. the value x such that $7x \equiv_{10} 1$.

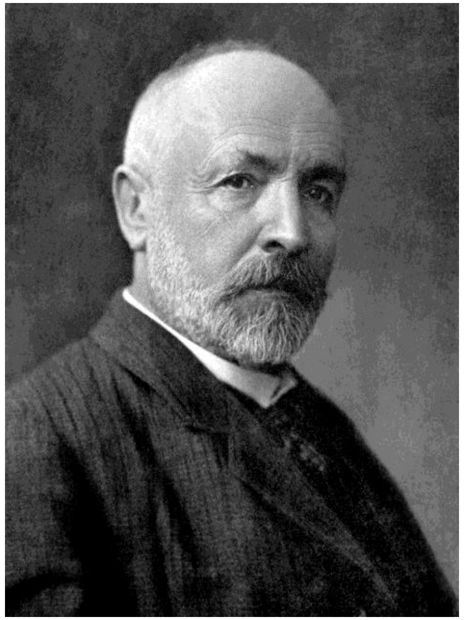
Set Theory

Motivation

Set theory is widely regarded as the foundation for all of mathematics.

In computing, there are applications in:

- Data Structures
- Databases
- Programming Languages



Father of Modern
Set Theory
Georg Cantor
(1845 – 1918)

Sets

Definition:

A **set** is an ____.

- We write ______________ to say that ______________________________.
- We write ______________ to say that ______________________________.

Set Notation

We’ll write a set as a collection of elements inside curly braces {}.

Sets are often given variable names with capital letters.

Common Sets

$\mathbb{R}$

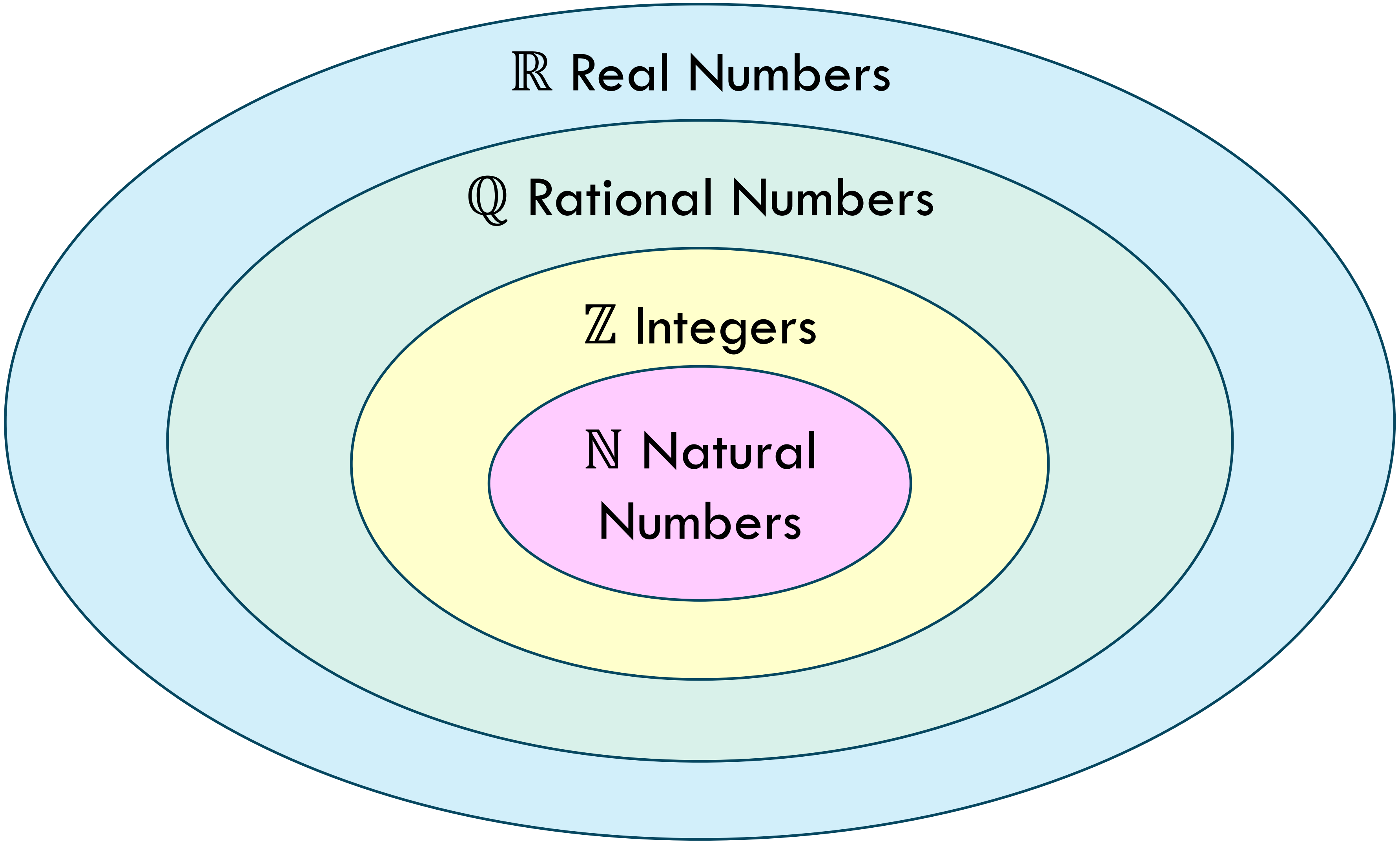
$\mathbb{Z}$

$\mathbb{N}$

$\mathbb{Q}$

$\emptyset$

Common Sets


ℝ Real Numbers
ℚ Rational Numbers
ℤ Integers
ℕ Natural Numbers

Sets can be elements of other sets

For example:

$A = \{\{1\}, \{2\}, \{1,2\}, \emptyset\}$

$B = \{1, 2\}$

Then ___________________________. And ___________________________.

Sets Builder Notation

Another way to describe a set is using set-builder notation.

$S = \{x : \mathrm{P}(x)\}$ means ____.

For example:

- $\{x \in \mathbb{Z} : x > 0\}$ is ____.

- $\{x \in \mathbb{N} : x \equiv_3 2\}$ is ____.

- $\left\{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\right\}$ is ____.

Set Cardinality

The cardinality of a set is ______________________________________.

The cardinality of a set A is often denoted ________________________.

What is the cardinality of the following sets?

- $A = \{x \in \mathbb{Z} \ : \ x \equiv_4 1 \text{ and } -10 \leq x \leq 10\}$
- $B = \emptyset$
- $C = \{\emptyset\}$

Relationships Between Sets

Set Equality

Sets A and B are equal if ______________________________.

In predicate logic, $A = B$ is defined as:

A = {1, 2, 3}
B = {3, 4, 5}
C = {3, 4}
D = {4, 3, 3}
E = {3, 4, 3}
F = {4, {3}}

Which sets are equal?

Subset

Set A is a subset of B if ______________________________.

In predicate logic, $A \subseteq B$ is defined as:

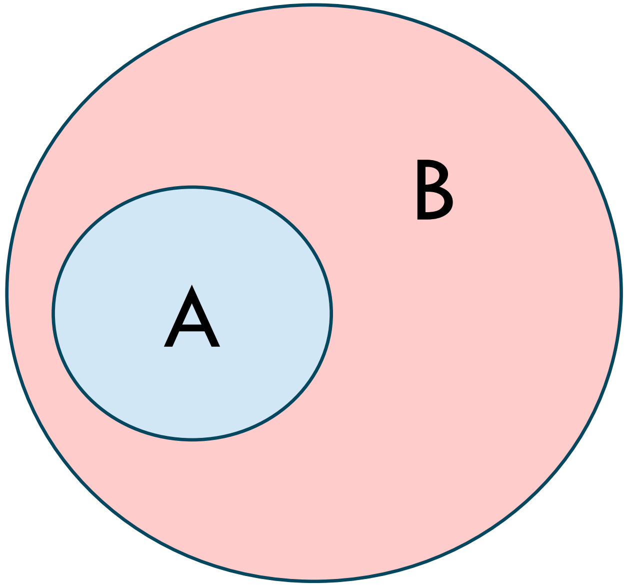
A = {1, 2, 3}
B = {3, 4, 5}
C = {3, 4}
D = {4, 3, 3}
E = {3, 4, 3}
F = {4, {3}}

Which sets are subsets?

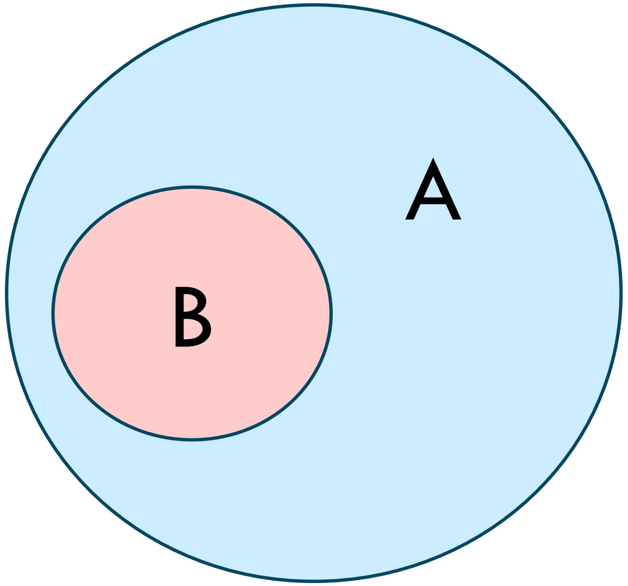
Set Equality and Subsets

$$A = B \equiv A \subseteq B \land B \subseteq A$$

A is a subset of B



B is a subset of A



$\in$ vs. $\subseteq$

$A = \{1, 2, 3\}$ $B = \{2\}$ $C = \{\emptyset, \{2\}\}$

- $\emptyset \subseteq A$?
- $\emptyset \in A$?
- $2 \subseteq B$?
- $2 \in B$?
- $B \in A$?
- $B \in C$?

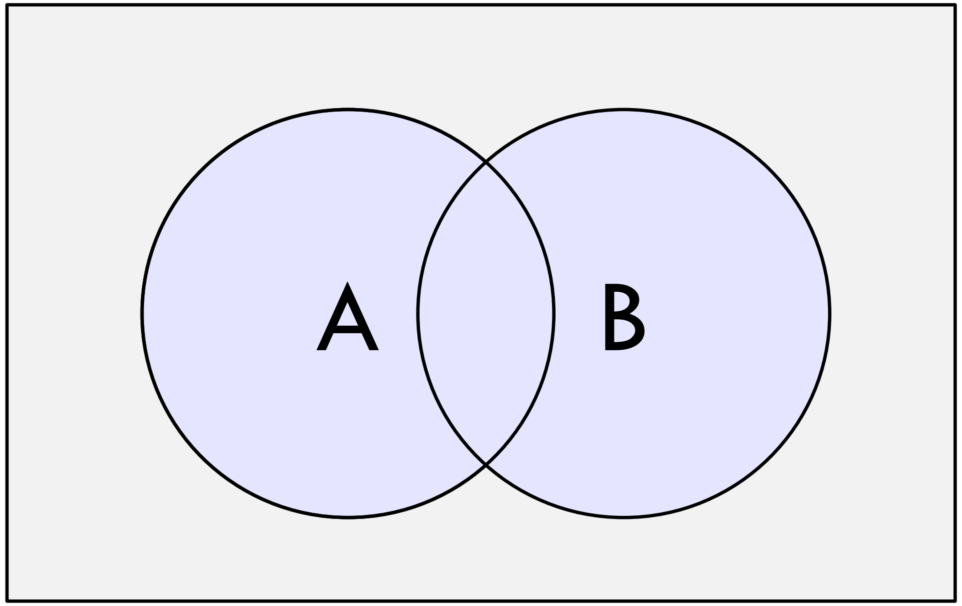
Set Operations

Combining Sets

Set Operations

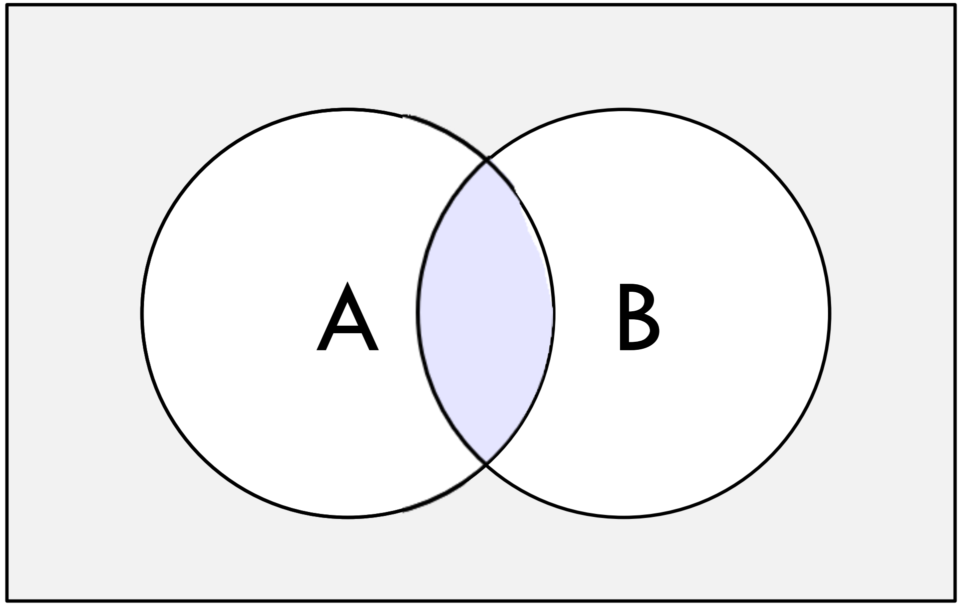
Union: $A \cup B$

$A \cup B = \{x : x \in A \vee x \in B\}$



Intersection: $A \cap B$

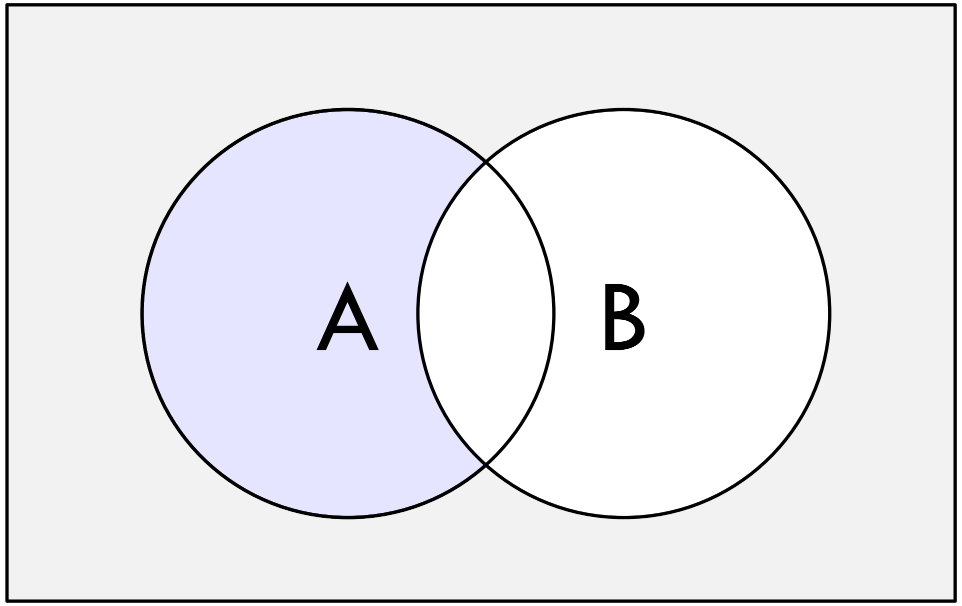
$A \cap B = \{x : x \in A \wedge x \in B\}$



Set Operations

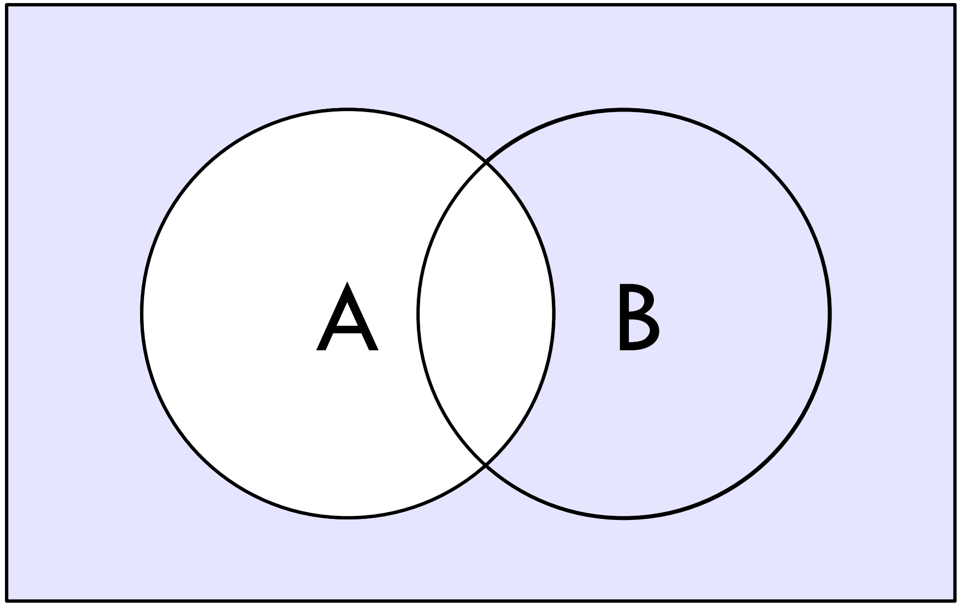
Set Difference: $A \setminus B$

$A \setminus B = \{x : x \in A \land x \notin B\}$



Set Complement: $\bar{A} = A^c$
(with respect to the universe $\mathcal{U}$)

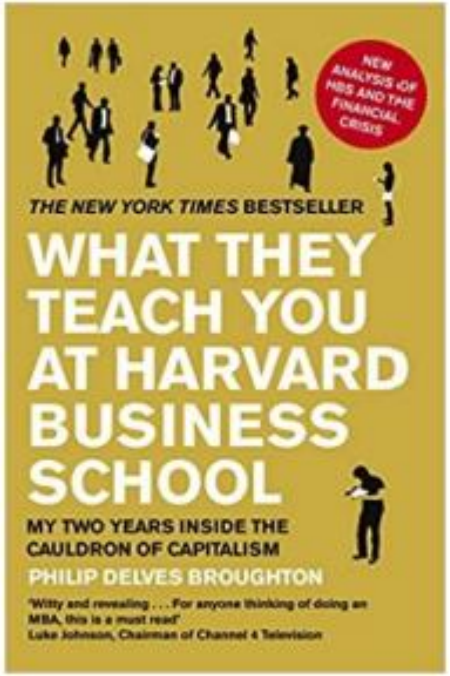
$\bar{A} = \{x \in \mathcal{U}: x \notin A\}$

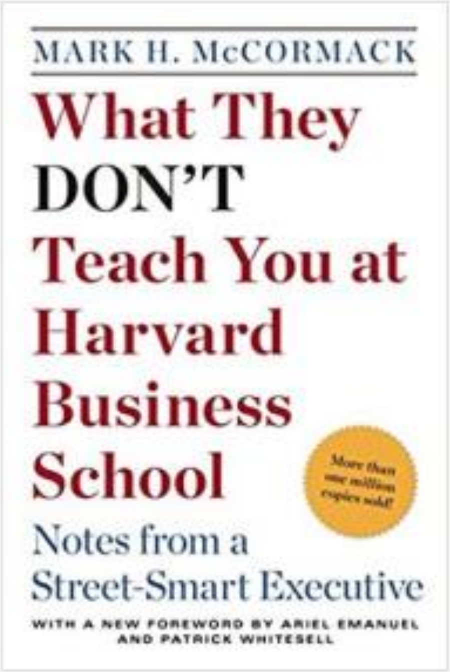


Set Operations

Erik Brynjolfsson
@erikbryn

It's remarkable that as recently as 11 years ago, the sum of all human knowledge could be provided in just two books.





Exercises

Definitions

$A \cup B = \{x : x \in A \lor x \in B\}$
$A \cap B = \{x : x \in A \land x \in B\}$
$A \setminus B = \{x : x \in A \land x \notin B\}$
$\bar{A} = \{x : x \notin A\}$

$A = \{1, 2, 3\}$ $B = \{3, 5, 6\}$ $C = \{3, 4\}$

Using only A, B, C and set operations, make the following sets. The universe is all integers.

- $\{1, 2, 3, 4, 5, 6\}$
- $\{3\}$
- $\{1, 2\}$
  See if you can do it without using set difference!

Powerset

Powerset: $\mathcal{P}\{A\}$

$\mathcal{P}(A) = \{X : X \subseteq A\}$

The powerset of A is the **set** of all subsets of A.

$\mathcal{P}(\{1,2\}) =$

$\mathcal{P}(\{a, b, c\}) =$

Cartesian Product

Cartesian Product: $A \times B$

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

The cartesian product of A with B is the set of ordered pairs of the form (a, b), where $a \in A$ and $b \in B$.

If $A = \{1,2\}$ and $B = \{a, b, c\}$ then:

$A \times B =$

$\mathbb{R} \times \mathbb{R} =$

Exercises

Compute the following:

$\{1,2\} \times \emptyset =$

$\mathcal{P}(\{2\} \times \{1, 3\}) =$

$\mathcal{P}(\{\emptyset\}) =$

$|\mathcal{P}(\{1, 2\}) \times \mathcal{P}(\{3, 4, 5\})| =$

Set Proofs

Two Claims

Determine if the following claims are true or false.

Claim 1: For all sets A, B, C, if $A \subseteq (B \cup C)$ then $A \subseteq B$ or $A \subseteq C$.

Claim 2: For all sets A, B, C it holds that $A \cap B \cap C \subseteq A \cup B$.

Claim 1

Claim 1: For all sets A, B, C, if $A \subseteq (B \cup C)$ then $A \subseteq B$ or $A \subseteq C$.

Claim 2

<u>Definition</u>
$A \subseteq B \equiv \forall x(x \in A \rightarrow x \in B)$

Claim 2: For all sets A, B, C it holds that $A \cap B \cap C \subseteq A \cup B$.

<u>Proof Strategy</u>

Claim 2

Definition
$A \subseteq B \equiv \forall x (x \in A \to x \in B)$

Claim 2: For all sets A, B, C it holds that $A \cap B \cap C \subseteq A \cup B$.

Proof

Anonymous Feedback

https://tinyurl.com/cse311feedback