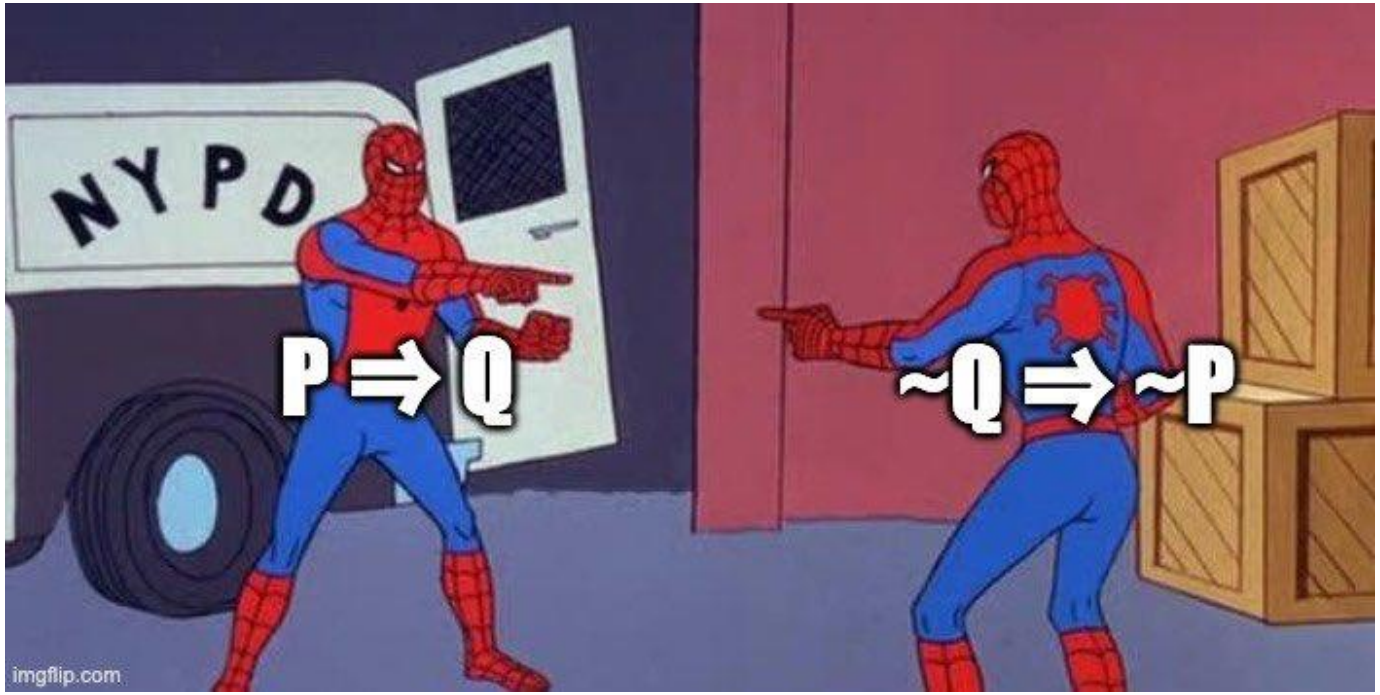
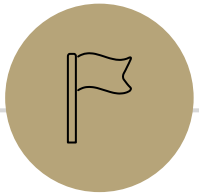


Proof by Contrapositive, Proof of Biconditional

CSE 311: Foundations of Computing I
Lecture 7

Announcements

- HW1 grades posted on Gradescope, printed copies of the solutions are at the front
- HW2 is due tonight at 11:59 pm



Review: Direct Proof

Direct Proof

Direct proof is one strategy for proving statements of the form $\forall x \left(\mathrm{P}(x) \rightarrow \mathrm{Q}(x)\right)$.

It involves:

- Taking an arbitrary x in the domain
- Assuming $\mathrm{P}(x)$ is true
- Proving that $\mathrm{Q}(x)$ is true

Direct Proof Example

Definitions
$\text{Odd}(x) \coloneqq \exists k(x = 2k + 1)$

Prove: "The product of two odd integers is odd."

$$\forall x \forall y \left(\left(\text{Odd}(x) \wedge \text{Odd}(y) \right) \rightarrow \text{Odd}(xy) \right)$$

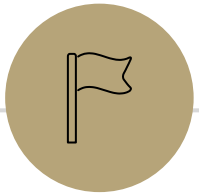
Let x and y be arbitrary integers. Suppose that x and y are odd. Then by definition of odd, there exists some integer k such that $x = 2k + 1$, and some integer j such that $y = 2j + 1$.

Then multiplying x and y, we can see that:

$$xy = (2k + 1) \cdot (2j + 1) = 4kj + 2j + 2k + 1 = 2(2kj + j + k) + 1$$

Since k, j are integers, $2kj + j + k$ is an integer. So xy is 2 times an integer plus 1. So by definition of odd, xy is odd.

Since x, y were arbitrary, we have shown that the product of two odd integers is odd.



Proof Strategy: Contrapositive

Proof by Contrapositive

Proof by contrapositive is another strategy for proving statements of the form _____.

The strategy is to _____.

Proof by Contrapositive

Definitions
$\text{Odd}(x) \coloneqq \exists k(x = 2k + 1)$

Prove: For an integer x, if $3x + 2$ is odd, then x is odd.

What's the claim in logic?

Try to prove this claim with a direct proof.

Proof by Contrapositive

Definitions
$\mathrm{Odd}(x) \coloneqq \exists k(x = 2k + 1)$

Prove: For an integer x, if $3x + 2$ is odd, then x is odd. $\forall x\big(\mathrm{Odd}(3x + 2) \to \mathrm{Odd}(x)\big)$

Proof by Contrapositive

How do we identify *when* to use a direct proof vs. a proof by contrapositive?

Try a direct proof first. If it seems challenging, then consider the contrapositive.

Another Proof by Contrapositive

Definitions
$\text{Even}(x) \coloneqq \exists k(x = 2k)$

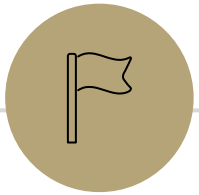
Prove by Contrapositive: For an integer n, if n^3 is even, then n is even.

Remark: Proof by Contrapositive

Just like we can show $p \to q$ is true by using a direct proof of $\neg q \to \neg p$, we can use our other logical equivalences.

Suppose for example the original claim is of the form $p \to (q \vee r)$. Then the contrapositive would be:

So the proof by contrapositive would be of the form:



Proof Strategy: Biconditional

Proof of a Biconditional

Recall that biconditionals are statements of the form:

_____.

The strategy is to prove such statements is to _____

_____.

Proof of a Biconditional

Prove: For an integer x, $2x + 3 = 15$ if and only if $x = 6$.

Remark: Biconditional Proofs

Each direction of the biconditional proof can use whichever proof type fits best (direct, contrapositive, etc.).

Consider the claim: For an integer n, $3n + 3$ is odd iff n is even.

Another Proof of a Biconditional

Definitions

$\text{Even}(x) \coloneqq \exists k(x = 2k)$

$\text{Odd}(x) \coloneqq \exists k(x = 2k + 1)$

Prove: For an integer n, $3n + 3$ is odd iff n is even.

Remark: Multiple Biconditionals

Suppose you wanted to prove $p \leftrightarrow q \leftrightarrow r$.

How many sub-proofs would you need?

Proof Strategies So Far

- Direct Proof
- Proof by Contrapositive
- Proof of Biconditional
- Proof by Cases
- Proof of Existence
- Disproof

Material for HW3 will be finished on Friday's lecture. The assignment will be posted tonight so that you can get started.