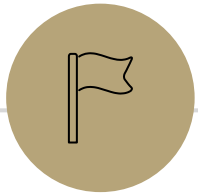


Predicate Logic, Nested Quantifiers

CSE 311: Foundations of
Computing I
Lecture 5

Announcements

- HW2
 - Due Wednesday, 11:59 pm
- Lecture on Monday



Review

Predicate Logic

3 Parts

1. Predicate – Function that outputs true or false.
 $\text{Prime}(x) := x \text{ is prime}$
2. Domain of Discourse – Set of possible inputs to a predicate.
E.g. Integers
3. Quantifiers – A statement about when a predicate is true
For all: \forall There exists: \exists

Translation

If a cat is fluffy, it is happy.

Domain of Discourse
Cats

Predicate Definitions
Fluffy(x) := x is fluffy
Happy(x) := x is happy

Translation

If a cat is fluffy, it is happy.

Domain of Discourse

Animals

Predicate Definitions

$\text{Cat}(x) := x$ is a cat

$\text{Fluffy}(x) := x$ is fluffy

$\text{Happy}(x) := x$ is happy



Nested Quantifiers

Example 1

Domain of Discourse
Mammals

Predicate Definitions

$\text{Walks}(x, y) := x \text{ walks } y$

$\text{Friends}(x, y) := x \text{ and } y \text{ are friends}$

$\text{Human}(x) := x \text{ is a human}$

$\text{Dog}(x) := x \text{ is a dog}$

Humans are not friends with each other.

Example 2

Domain of Discourse
Mammals

Predicate Definitions

$\text{Walks}(x, y) := x \text{ walks } y$

$\text{Friends}(x, y) := x \text{ and } y \text{ are friends}$

$\text{Human}(x) := x \text{ is a human}$

$\text{Dog}(x) := x \text{ is a dog}$

All humans are friends with the dogs that they walk.

Example 3

Domain of Discourse
Mammals

Predicate Definitions

$\text{Walks}(x, y) := x \text{ walks } y$

$\text{Friends}(x, y) := x \text{ and } y \text{ are friends}$

$\text{Human}(x) := x \text{ is a human}$

$\text{Dog}(x) := x \text{ is a dog}$

Every human walks a dog.

Example 3

Domain of Discourse
Mammals

Predicate Definitions

$\text{Walks}(x, y) := x$ walks y

$\text{Friends}(x, y) := x$ and y are friends

$\text{Human}(x) := x$ is a human

$\text{Dog}(x) := x$ is a dog

Every human walks a dog.

a) $\forall x \exists y (\text{Human}(x) \wedge \text{Dog}(y) \wedge \text{Walks}(x, y))$

b) $\forall x \exists y \left((\text{Human}(x) \wedge \text{Dog}(y)) \rightarrow \text{Walks}(x, y) \right)$

c) $\forall x \exists y \left(\text{Human}(x) \rightarrow (\text{Dog}(y) \wedge \text{Walks}(x, y)) \right)$

d) $\forall x \exists y \left(\text{Human}(x) \wedge (\text{Dog}(y) \rightarrow \text{Walks}(x, y)) \right)$

Poll Everywhere
pollev.com/anjalia

Example 4

Domain of Discourse
Mammals

Predicate Definitions

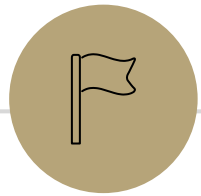
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$\text{Friends}(x, y) := x \text{ and } y \text{ are friends}$

$\text{Human}(x) := x \text{ is a human}$

$\text{Dog}(x) := x \text{ is a dog}$

Every human walks exactly one dog.



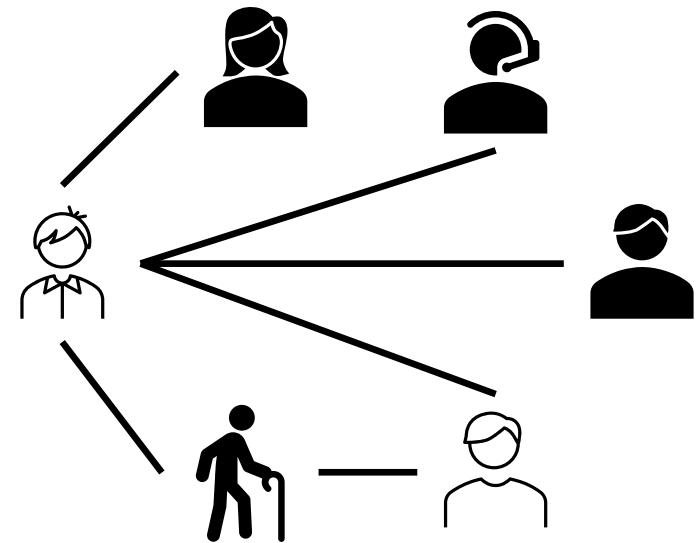
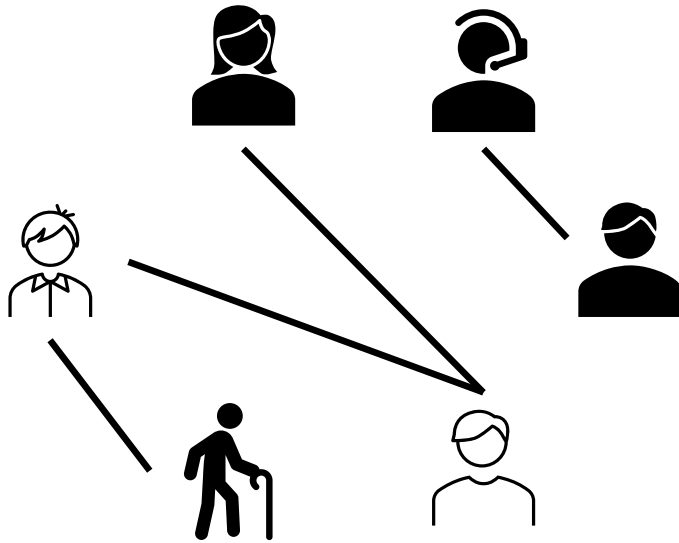
Quantifier Order

Quantifier Order

Translate to logic. The domain of discourse is people. The predicate $\text{Friends}(x, y)$ is defined as x and y are friends.

Everyone is friends with someone.

Someone is friends with everyone.



Quantifier Order

$\forall x \exists y P(x, y)$ means _____

$\exists y \forall x P(x, y)$ means _____

Quantifier Order

$\forall x \exists y \text{ Likes}(x, y)$

$\forall y \exists x \text{ Likes}(x, y)$

$\exists x \forall y \text{ Likes}(x, y)$

$\exists y \forall x \text{ Likes}(x, y)$

Quantifier Order

Let our domain of discourse be $\{A, B, C, D, E\}$

And our proposition $P(x, y)$ be given by the table.

What should we look for in the table?

$$\exists x \forall y P(x, y)$$

$$\forall x \exists y P(x, y)$$

	y				
$P(x, y)$	A	B	C	D	E
A	T	T	T	T	T
B	T	F	F	T	F
C	F	T	F	F	F
D	F	F	F	F	T
E	F	F	F	T	F

Quantifier Order

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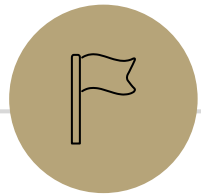
$$\exists x \forall y P(x, y)$$

A row, where every entry is T

$$\forall x \exists y P(x, y)$$

In every row there must be a T

	y				
$P(x, y)$	A	B	C	D	E
A	T	T	T	T	T
B	T	F	F	T	F
C	F	T	F	F	F
D	F	F	F	F	T
E	F	F	F	T	F



Negating Quantifiers

DeMorgan's Law for Quantifiers

Consider the following sentences:

- There does not exist a green penguin.
- Every penguin is a color other than green.

Are they logically equivalent?

DeMorgan's Law for Quantifiers

Consider the following sentences:

- Not every person can dance.
- There is a person that cannot dance.

Are they logically equivalent?

DeMorgan's Law for Quantifiers

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

I.e. to negate an expression with a quantifier:

1. Switch the quantifier (\forall becomes \exists , and vice versa).
2. Negate the expression inside.

Example 1

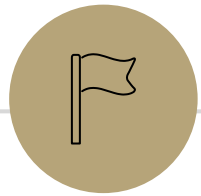
Translate to predicate logic & rewrite using DeMorgan's Law.

There is no integer which is prime and even.

Example 2

Translate to predicate logic & rewrite using DeMorgan's Law.

There is no integer greater than or equal to every other integer.



Predicate Logic Equivalence

Motivation

- We saw with the last two examples that there may be different predicate logic expressions that have the same meaning
- We can prove logical equivalence of Predicate Logic statements like we did for Propositional Logic
- Same equivalence rules still apply, in addition to DeMorgan's Law for Quantifiers

Proving Predicate Logic Equivalence

"No odd integer is equal to an even integer."

Alice translated this as: _____

Bob translated this as: _____

Prove that these translations are logically equivalent.

Proving Predicate Logic Equivalence

$$\neg \exists x \exists y (\text{Odd}(x) \wedge \text{Even}(y) \wedge (x = y))$$

$$\equiv \forall x \forall y \left((\text{Odd}(x) \wedge \text{Even}(y)) \rightarrow (x \neq y) \right)$$

Proving Predicate Logic Equivalence

$$\neg \forall x (P(x) \rightarrow \exists y Q(x, y))$$

$$\equiv \exists x \forall y (P(x) \wedge \neg Q(x, y))$$

Anonymous Feedback

<https://tinyurl.com/cse311feedback>

