CSE 311: Foundations of Computing I

Practice Final Part 2

Name: ______________________________

UW ID: ______________________________

Instructions:

- You have **1 hour** to complete the exam.
- There are 3 problems on this exam, totaling 65 points.
- The exam is closed book. You may not use cell phones or calculators. You may only use the reference sheets provided.
- All answers you want graded should written on the exam paper.
- If you need extra space, use the back of a page.

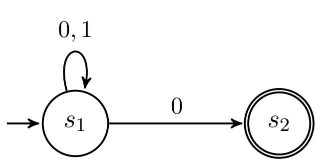
1. True or False [15 points]

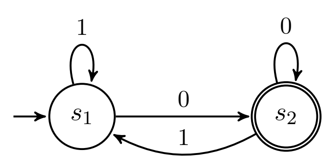
For the following questions, determine whether the statement is true or false. Then provide 1-3 sentences of explanation. Your explanations **do not** need to be full or formal proofs.

(a) (5 points) Any subset of a regular language is also regular.

(b) (5 points) There exists a language L such that there is some CFG that can generate L, but no NFA can recognize L.

(c) (5 points) The following NFAs recognize the same language.





2. Models of Computation [30 points]

Consider the language of binary strings where the number of 0's minus the number of 1's is divisible by 3.

(a) (10 points) Construct a Regular Expression for this language.

(b) (10 points) Construct a DFA for this language.

(c) (10 points) Construct a CFG for this language.

3. Structural Induction [20 points]
Let the alphabet be $\Sigma = \{0, 1, 2, ..., 9\}$. Consider the following recursive definition of the set S of strings over Σ:

Basis Step: $\varepsilon \in S$
Recursive Step: If $w \in S$ and $a \in \Sigma$ then $wa \in S$

Consider the following recursively-defined functions `double` and `sum`. Here the `int` function simply accepts a character in Σ and returns the integer value of that character.

$\texttt{double}(\varepsilon) = \varepsilon$
$\texttt{double}(wa) = \texttt{double}(w)aa$ for $w \in S, a \in \Sigma$

$\texttt{sum}(\varepsilon) = 0$
$\texttt{sum}(wa) = \texttt{sum}(w) + \texttt{int}(a)$ for $w \in S, a \in \Sigma$

Prove by structural induction that $\texttt{sum}(\texttt{double}(w)) \equiv_2 0$ for all $w \in S$.