CSE 311: Foundations of Computing I

Practice Final Part 1

Name: ______________________________________

UW ID: ______________________________________

Instructions:

- You have **1 hour** to complete the exam.
- There are 3 problems on this exam, totaling 70 points.
- The exam is closed book. You may not use cell phones or calculators. You may only use the reference sheets provided.
- All answers you want graded should written on the exam paper.
- If you need extra space, use the back of a page.

1. True or False [30 points]

For the following questions, determine whether the statement is true or false. Then provide 1-3 sentences of explanation. Your explanations **do not** need to be full or formal proofs.

(a) (5 points) "p only if q" and "q is necessary for p", are both best translated as $p \to q$.

(b) (5 points) One way to prove that $p \to q$ is true is to show that the converse, $q \to p$, is false.

(c) (5 points) The implication $\forall y \exists x$ P$(x, y) \to \exists x \forall y$ P(x, y) is true regardless of what the predicate P is.

(d) (5 points) Suppose a, b, m, n are all integers greater than 1. If $a \equiv_m b$ and $m \mid n$, then $a \equiv_n b$.

(e) (5 points) Suppose A, B are sets. If $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$.

(f) (5 points) Strong induction proofs always require more than one base case.

2. Contradiction [20 points]

Recall that we defined the rational numbers as the set $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$. For example, $\frac{5}{8}, -12$ are rational.

We defined the irrational numbers as all real numbers that are not rational. For example, $\sqrt{2}, \pi$ are irrational.

Prove by contradiction that for all real numbers x, y if x is rational and xy is irrational, then y is irrational.

3. Induction [20 points]
Consider the following recursive definition of a function f defined over all $n \in \mathbb{N}$:

$$\begin{aligned} f(n) &= 1 && \text{if } n = 0 \\ f(n) &= 3 \cdot f(n-1) && \text{if } 1 \leq n \leq 2 \\ f(n) &= 3 \cdot f(n-1) - 2 \cdot f(n-2) + 6 \cdot f(n-3) && \text{for all } n > 2 \end{aligned}$$

Prove using strong induction that, for all $n \in \mathbb{N}$, we have $f(n) = 3^n$.