CSE 311: Foundations of Computing I

Homework 5 (due Wednesday, July 26th at 11:59 PM)

Directions: *Write up carefully argued solutions to the following problems. Each solution should be clear enough that it can explain why it works to someone who does not already understand the answer. If you work with others, remember to follow the collaboration policy outlined in the syllabus. Be sure to read the Typesetting and Grading guidelines prior to submitting.*

Important Note: *Grades for this assignment will be released on Thursday June 27th, the day before the CSE 311 midterm. If you choose to use Late Days, you may not receive feedback before the midterm occurs.*

1. Set Computation (12 points)

Compute each of the following sets. We are given that $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and the universe is all integers.

You do not need to show any work; only your final solution. Recall that the order in which you list the elements of a set does not matter.

(a) [3 Points] $A \times (\varnothing \cup \{7\})$

(b) [3 Points] $\mathcal{P}(\mathcal{P}(\varnothing))$

(c) [3 Points] $(A \times B) \cap (B \times A)$

(d) [3 Points] $\mathcal{P}(A \setminus \overline{B})$

2. Set Equality (16 points)

Prove that for all sets A, B, and C we have: $(B \setminus A) \cup (C \setminus A) = (B \cup C) \setminus A$. You may use a chain of equivalence proof OR a subset proof in each direction.

3. We've Got The Power (24 points)

Prove or disprove the following claims. For each claim, please specify clearly if you are writing a proof or a disproof.

(a) [12 Points] For all sets S and T: $\mathcal{P}(S \cap T) \subseteq \mathcal{P}(S) \cap \mathcal{P}(T)$.

(b) [12 Points] For all sets S and T: $\mathcal{P}(S \cup T) \subseteq \mathcal{P}(S) \cup \mathcal{P}(T)$.

4. Keeping up with the Cartesians (15 points)

(a) [12 Points] Prove that for all sets A, B, C if A is non-empty and $A \times B = A \times C$, then $B = C$.

Hint: Write a subset proof in each direction.

(b) [3 Points] Does the claim in part a) still hold if A is empty? Why or why not?

5. Induction I (16 points)

Prove by induction that for every $n \in \mathbb{N}$, the following equality is true:

$$0 \cdot 2^0 + 1 \cdot 2^1 + 2 \cdot 2^2 + \cdots + n \cdot 2^n = (n-1)2^{n+1} + 2.$$

6. Induction II (16 points)

Prove by induction that $5 \mid (6^n - 1)$ for all $n \in \mathbb{N}$.

7. Feedback (2 points)

Please share approximately how many hours you spent working on this assignment. Report your estimate to the nearest hour. This will help us calibrate our assignments in the future.

If you have any additional feedback, we welcome that as well.