CSE 311: Foundations of Computing I

Homework 4 (due Wednesday, July 19th at 11:59 PM)

Directions: *Write up carefully argued solutions to the following problems. Each solution should be clear enough that it can explain why it works to someone who does not already understand the answer. If you work with others, remember to follow the collaboration policy outlined in the syllabus. Be sure to read the Typesetting and Grading guidelines prior to submitting.*

1. Euclid's Algorithm (12 points)

Compute the following using Euclid's Algorithm. Show your intermediate results as a sequence of $\gcd()$ calls.

(a) [4 Points] $\gcd(297, 129)$

(b) [4 Points] $\gcd(354, 123)$

(c) [4 Points] $\gcd(7^{49} + 1, 7)$

2. Divided Up (12 points)

Let x, y, z be arbitrary integers such that $x|y$ and $y|z$.

(a) [6 Points] Prove or disprove: $xy|z$. (Please specify in your solution if you are writing a proof or a disproof.)

(b) [6 Points] Prove or disprove: $x|z$. (Please specify in your solution if you are writing a proof or a disproof.)

3. Oddly Divisible (16 points)

Prove that for all odd integers k, the statement $8 \mid (k^2 - 1)$ holds.

4. Mod Madness (20 points)

(a) [12 Points] Prove the claim: For all integers a, b and all positive integers c, m, it holds that $a \equiv_m b$ if and only if $ac \equiv_{mc} bc$.

(b) [8 Points] Disprove the claim: For all integers a, b and all positive integers c, m, if $ac \equiv_m bc$ then $a \equiv_m b$.

5. Fair and Square (16 points)

Prove that for all integers n, $n^2 \equiv_4 0$ or $n^2 \equiv_4 1$.

6. Feedback (2 points)

Please share approximately how many hours you spent working on this assignment. Report your estimate to the nearest hour. This will help us calibrate our assignments in the future.

If you have any additional feedback, we welcome that as well.