CSE 311: Foundations of Computing I

Homework 3 (due Wednesday, July 12th at 11:59 PM)

Directions: *Write up carefully argued solutions to the following problems. Each solution should be clear enough that it can explain why it works to someone who does not already understand the answer. If you work with others, remember to follow the collaboration policy outlined in the syllabus. Be sure to read the Typesetting and Grading guidelines prior to submitting.*

1. Predicate Logic Equivalence (8 points)

Prove the following assertion using a sequence of logical equivalences.

$$\neg\exists x\forall y(\mathsf{P}(x) \land (\mathsf{Q}(y) \rightarrow \mathsf{Q}(x))) \equiv \forall x\exists y(\mathsf{P}(x) \rightarrow (\mathsf{Q}(y) \land \neg\mathsf{Q}(x)))$$

2. Being Direct (12 points)

(a) [4 Points] Let the domain of discourse be integers. Define the predicates $\mathsf{Odd}(x) := \exists k(x = 2k + 1)$, and $\mathsf{Even}(x) := \exists k(x = 2k)$.

Translate the following claim to predicate logic:

For all odd integers n and m, $3n + m$ is even.

(b) [8 Points] Prove that the claim is true.

3. Are you (Contra)positive? (16 points)

Two integers are said to have the **same parity** if they are both odd or both even.

Prove that for all integers x and y, if $x + y$ is even, then x and y have the same parity.

Hint: Prove by contrapositive. Within your contrapositive proof, you may wish to include two separate cases.

4. Oddly Even (16 points)

Prove that for all integers n, $n + 5$ is odd if and only if n^2 is even.

Hint: To prove the biconditional, you will need to write a proof of the implication in each direction. Within the proof of one of the directions, you may find it helpful to use another proof strategy (contrapositive, cases, etc.)

5. Disprove! (8 points)

Disprove the following claims.

(a) [4 Points] For all integers x and y, if $x^2 = y^2$ then $x = y$.

(b) [4 Points] If x and y are two prime numbers greater than 19, then $|x - y| > 2$.
Note: You may state that an integer is prime without proving it.

6. The Case(s) of the Invisible Ghost (16 points)

Suppose you are a character in a horror movie. In this movie, there is an invisible ghost in your house and in order to get rid of it, you need to turn on the light in the room it is in. However, because this is a horror movie, for plot reasons you are only allowed to turn on the light in one room at a time and only for a split second. If you fail to find the ghost after 4 tries, then the ghost eats your homework. To make the movie more suspenseful, every time you flash a light, the ghost always moves to a neighboring room. Your house is laid out as line of four rooms R_1, R_2, R_3, R_4. For example, if the ghost starts in R_2, and you first check R_3, then immediately after, the ghost could move to either R_1 or R_3. However, if the ghost had been in a room on the end, like R_1, then it could only move to R_2.

(a) [6 Points] Prove using cases that if you check R_2 and then R_3, then either you have caught the ghost or it is in R_1 or R_3.

(b) [6 Points] Prove using cases that if the ghost is currently in R_1 or R_3 then if you check R_3 and then R_2, then you always catch the ghost.

(c) [4 Points] Use parts (a) and (b) to find a sequence of 4 checks which always catches the ghost. Give a 1-2 sentence explanation justifying why your answer is correct.

7. Feedback (2 points)

Please share approximately how many hours you spent working on this assignment. Report your estimate to the nearest hour. This will help us calibrate our assignments in the future.

If you have any additional feedback, we welcome that as well.