

CSE 311: Foundations of Computing I

Homework 2 (due Wednesday, July 5th at 11:59 PM)

Directions: Write up carefully argued solutions to the following problems. Each solution should be clear enough that it can explain why it works to someone who does not already understand the answer. If you work with others, remember to follow the collaboration policy outlined in the syllabus. Be sure to read the Typesetting and Grading guidelines prior to submitting.

1. Normal Forms (8 points)

Consider the following function F :

| p | q | r | $F(p, q, r)$ |
|-----|-----|-----|--------------|
| T | T | T | F |
| T | T | F | F |
| T | F | T | T |
| T | F | F | F |
| F | T | T | T |
| F | T | F | F |
| F | F | T | F |
| F | F | F | T |

- (a) [4 Points] Write a propositional logic expression for F in DNF form (ORs of ANDs).
- (b) [4 Points] Write a propositional logic expression for F in CNF form (ANDs of ORs).

2. Logical Equivalence (16 points)

Prove the following assertions using a sequence of logical equivalences.

Hint: For equivalences where one side is much longer than the other, a good heuristic is to start with the longer side and try to apply the rules that will shorten it.

Hint for (d): if you find yourself wanting to transform $\neg T$ into F, you may want to back up and try another approach to solving the problem. It is possible to prove $\neg T \equiv F$ using equivalences, but it takes a bit of work to do so.

- (a) [4 Points] $P \rightarrow (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$
- (b) [4 Points] $(\neg P \rightarrow Q) \wedge (Q \rightarrow P) \equiv P$
- (c) [4 Points] $(P \rightarrow Q) \vee (P \rightarrow \neg Q) \equiv T$
- (d) [4 Points] $((P \vee \neg P) \rightarrow P) \wedge (Q \rightarrow P) \equiv P$

3. Fun with Notation (8 points)

Consider the following proposition: $((\neg p) \wedge (p \vee q)) \wedge \neg \neg q$

- (a) [4 Points] Write the proposition in Boolean Algebra notation. Do not simplify the expression.
- (b) [4 Points] Write the proposition as a Digital Circuit.

4. Pet Project (16 points)

Let the domain of discourse be people and pets. Define the predicates $\text{Person}(x)$ to mean that x is a person and $\text{Pet}(y)$ to mean that y is a pet. Define the predicate $\text{TakesCareOf}(x, y)$ to mean that x takes care of y and the predicate $\text{HasPaws}(y)$ to mean that y has paws.

Translate each of the following logical statements into English. You should not simplify. However, you should use the techniques shown in lecture for producing more natural translations when restricting domains and for avoiding the introduction of variable names when not necessary.

- (a) [4 Points] $\neg \exists x (\text{Person}(x) \wedge \text{HasPaws}(x))$
- (b) [4 Points] $\exists x (\text{Person}(x) \wedge \exists y (\text{Pet}(y) \wedge \text{TakesCareOf}(x, y) \wedge \text{TakesCareOf}(y, x)))$
- (c) [4 Points] $\exists x (\text{Person}(x) \wedge \exists y (\text{Pet}(y) \wedge \text{TakesCareOf}(x, y) \wedge \neg \text{HasPaws}(y)))$
- (d) [4 Points] $\forall x (\text{Pet}(x) \rightarrow \exists y ((\text{Person}(y) \wedge \text{TakesCareOf}(y, x)))$

5. Roaring Lions (16 points)

Let the domain of discourse be lions. We define the predicate $\text{Respects}(x, y)$ to mean that x respects y . We define the predicate $\text{Sprints}(x)$ to mean that x sprints. We define the predicate $\text{Roars}(x)$ to mean that x roars. You can also assume an “=” operator that is true when x and y are the same lion, and an “ \neq ” operator that is true when x and y are different lions.

Translate each of the following English statements into predicate logic. Do not simplify.

- (a) [4 Points] There is a lion that every lion respects.
- (b) [4 Points] All lions that roar respect a lion that does not roar.
- (c) [4 Points] There is a lion that respects itself, but does not respect any other lion.
- (d) [4 Points] There is exactly one lion that sprints.

6. Mind Your Ps and Qs (8 points)

Consider the following two expressions:

$$\forall x (\text{P}(x) \vee \text{Q}(x)) \quad \forall x (\text{P}(x)) \vee \forall x (\text{Q}(x))$$

- (a) [4 Points] Give a domain of discourse and definitions of P and Q such that these expressions are **not** equivalent. Explain why your examples work (1-2 sentences).
- (b) [4 Points] Give a domain of discourse and definitions of P and Q such that these expressions **are** equivalent. Explain why your examples work (1-2 sentences).

7. Feedback (2 points)

Please share approximately how many hours you spent working on this assignment. Report your estimate to the nearest hour. This will help us calibrate our assignments in the future.

If you have any additional feedback, we welcome that as well.