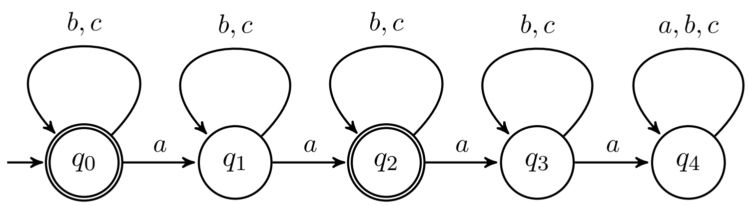
Quiz Section 9: Minimization, NFAs, Subset Construction, Irregularity – Solutions

Task 1 – DFAs & Minimization

Minimize the following DFA.

For each step of the algorithm, write down the groups of states, which group was split in that step and the reason for splitting that group. At the end, write down the minimized DFA, with each state named by the set of states of the original machine that it represents (e.g., "B, C" if it represents B and C).



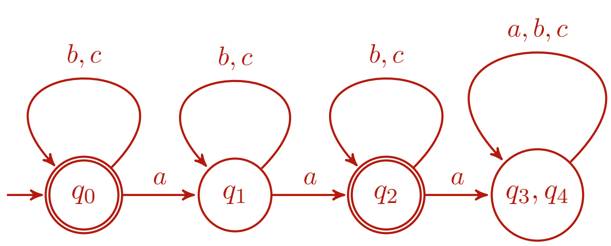
Step 1: q_0, q_2 are final states and the rest are not final. So, we start with the initial partition with the following groups: group 1 is $\{q_0, q_2\}$ and group 2 is $\{q_1, q_3, q_4\}$.

Step 2: q_1 is sending a to group 1 while q_3, q_4 are sending a to group 2. So, we divide group 2. We get the following groups: group 1 is $\{q_0, q_2\}$, group 3 is $\{q_1\}$ and group 4 is $\{q_3, q_4\}$.

Step 3: q_0 is sending a to group 3 and q_2 is sending a to group 4. So, we divide group 1. We will have the following groups: group 3 is $\{q_1\}$, group 4 is $\{q_3, q_4\}$, group 5 is $\{q_0\}$ and group 6 is $\{q_2\}$.

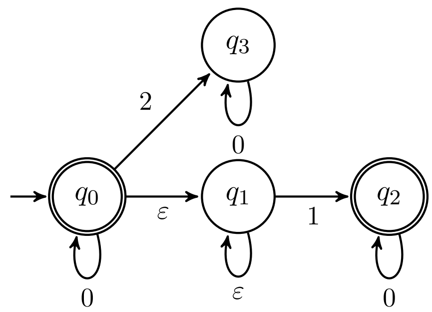
At this point we are done since the only group consisting of more than one state is group $\{q_3, q_4\}$ which will not be split since for every symbol every transition from either q_3 or q_4 goes back to group $\{q_3, q_4\}$.

The minimized DFA is the following:



Task 2 – NFAs

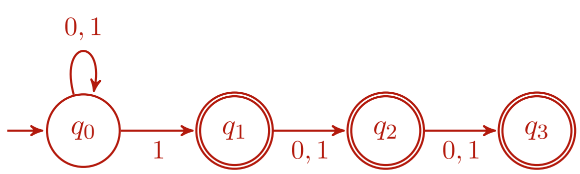
a) What language does the following NFA accept?



All strings of only 0's and 1's not containing more than one 1.

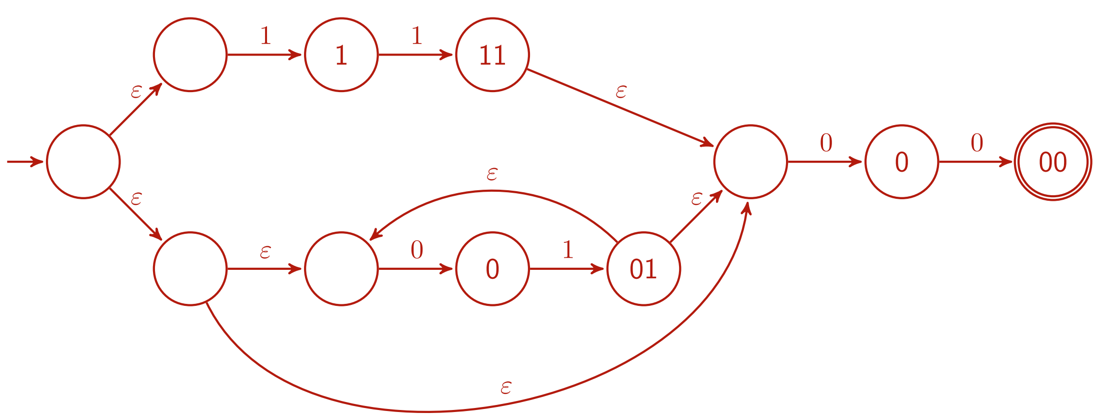
b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

The following is one such NFA:



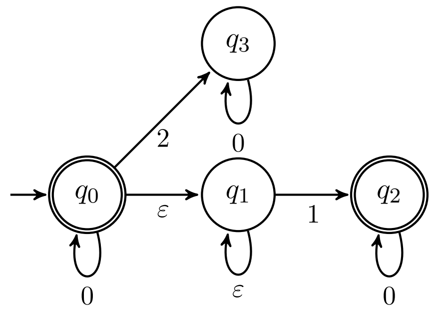
Task 3 – RE to NFA

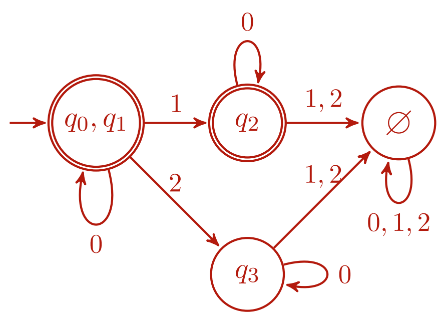
Convert the regular expression "$(11 \cup (01)^*)00$" to an NFA using the algorithm from lecture. You may skip adding ε-transitions for concatenation if they are obviously unnecessary, but otherwise, you should *precisely* follow the construction from lecture.



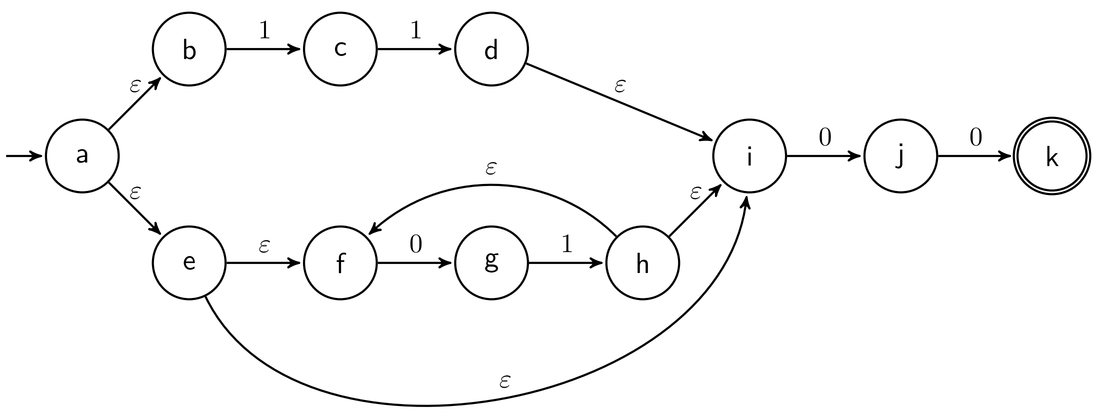
Task 4 – NFAs to DFAs

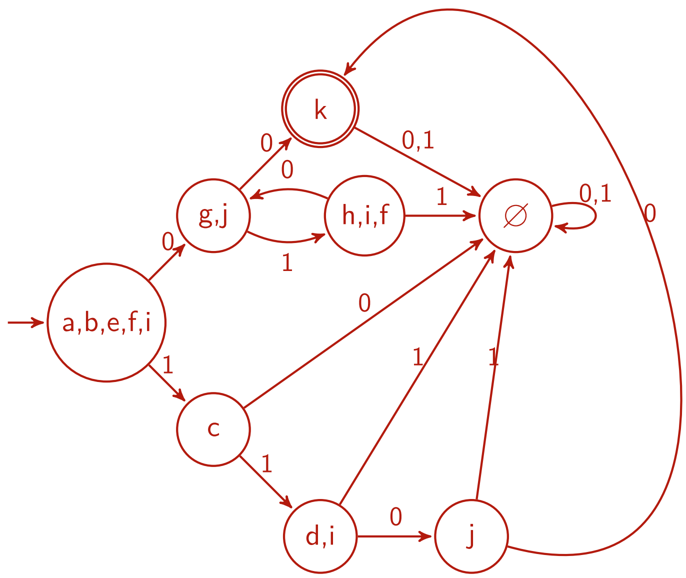
a) Convert the following NFA to a DFA for the same language:





b) Convert the following NFA to a DFA for the same language:





Task 5 – Irregularity

a) Let $\Sigma = \{0,1\}$. Prove that $\{0^n1^n0^n \ : \ n \geqslant 0\}$ is not regular.

Let $L = \{0^n1^n0^n \ : \ n \geqslant 0\}$. Let D be an arbitrary DFA, and suppose for contradiction that D accepts L. Consider $S = \{0^n1^n : n \geqslant 0\}$. Since S contains infinitely many strings and D has a finite number of states, two strings in S must end up in the same state. Say these strings are 0^i1^i and 0^j1^j for some $i, j \geqslant 0$ such that $i \neq j$. Append the string 0^i to both of these strings. The two resulting strings are:

$a = 0^i1^i0^i$ Note that $a \in L$.

$b = 0^j1^j0^i$ Note that $b \notin L$, since $i \neq j$.

Since a and b end up in the same state, but $a \in L$ and $b \notin L$, that state must be both an accept and reject state, which is a contradiction. Since D was arbitrary, there is no DFA that recognizes L, so L is not regular.

b) Let $\Sigma = \{0, 1, 2\}$. Prove that $\{0^n(12)^m \ : \ n \geqslant m \geqslant 0\}$ is not regular.

Let $L = \{0^n(12)^m \ : \ n \geqslant m \geqslant 0\}$. Let D be an arbitrary DFA, and suppose for contradiction that D accepts L. Consider $S = \{0^n : n \geqslant 0\}$. Since S contains infinitely many strings and D has a finite number of states, two strings in S must end up in the same state. Say these strings are 0^i and 0^j for some $i, j \geqslant 0$ such that $i > j$. (We know that the two strings must be different so one of them must have larger length and we have just chosen to call the larger length i.) Append the string $(12)^i$ to both of these strings. The two resulting strings are:

$a = 0^i(12)^i$ Note that $a \in L$.

$b = 0^j(12)^i$ Note that $b \notin L$, since $i > j$.

Since a and b end up in the same state, but $a \in L$ and $b \notin L$, that state must be both an accept and reject state, which is a contradiction. Since D was arbitrary, there is no DFA that recognizes L, so L is not regular.

Note: We couldn't have appended $(12)^j$ because in that case both $0^j(12)^j$ and $0^i(12)^j$ would be in L since $j < i$.