Quiz Section 1: Propositional Logic Translation – Solutions

Review

Task 1 – Warm-Up

Translate the English sentences below into symbolic logic.

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Since we're in "if...then..." form, the sentence is an implication.
a: I am lifting weights
b: I do a warm-up exercise

$$a \to b$$

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

a: I am cold
b: I am going to bed
c: I am two-years old
d: I carry a blanket

$$[(a \wedge b) \vee c] \to d$$

How did we know the translation wasn't $[a \wedge (b \vee c)] \to d]$? Two hints were available: first, omitted words ("going to bed" instead of "I am going to bed" indicates b should be closer to the "and" than the "or"), second the interpretation of the sentence – two-year olds more commonly carry blankets during the day than warm adults.

Task 2 – If I can translate, then...

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

a) whenever I walk my dog, I make new friends.

p: I walk my dog
r: I make new friends

$$p \to r$$

The promise is that we will definitely make new friends on the condition of walking our dog.

b) I will drink coffee, if Starbucks is open or my coffeemaker works.

a: I will drink coffee
b: Starbucks is open
c: my coffeemaker works

$$(b \lor c) \to a$$

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

a: One is a U.S. citizen
b: One is over 18
c: One is eligible to vote

$$(a \land b) \to c$$

The original sentence omits a subject. We introduced a dummy subject "one" to the propositions, you might have said "someone" or "a person" instead (among other options).

d) I can go home only if I have finished my homework.

p: I can go home.
r: I have finished my homework.

$$p \to r$$

The promise here is that if I can go home then I must have finished my homework. It can sometimes help to imagine when the sentence is broken. Is it broken if my homework is finished, but I cannot go home? No, perhaps I also have to say bye to my friends before I leave. But if I can go home with unfinished homework, then the promise is broken.

"Only if" is one of the more confusing arrangements – the consequence ("the then part") is adjacent to the "only if."

e) Having an internet connection is necessary to log onto zoom.

p: One has an internet connection
r: One can log onto zoom

$$r \to p$$

The internet connection is not enough (what if you don't have the meeting link?) but certainly if you are in the meeting then you have a connection.

Task 3 – I can rewrite these formulas in English, only if...

Given propositions and a logical formula, write **two** potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.

a) p: The sun is out
r: We have class outside

$$p \to r$$

If the sun is out, then we have class outside.
Whenever the sun is out, we have class outside.

b) a: the book has been out for a week.
b: I don't have homework.
c: I have finished reading the book.

$$(a \wedge b) \rightarrow c$$

I have finished reading the book, if it has been out for a week and I don't have homework.
The book being out for a week and me not having homework is sufficient for me to have finished reading the book.

c) p: I have read the manual
r: I operate the machine

$$r \rightarrow p$$

I operate the machine only if I have read the manual.
Operating the machine implies that I have read the manual.

Task 4 – Translation

For each of the following, define propositional variables and translate the sentences into logical notation.

a) I will remember to send you the address only if you send me an e-mail message.

p: I will remember to send you the address.
r: You send me an e-mail message.

$$p \rightarrow r$$

b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

a: Berries are ripe along the trail.
b: Hiking is safe.
c: Grizzly bears have not been seen in the area.

$$a \rightarrow (b \leftrightarrow c)$$

c) Unless I am trying to type something, my cat is either eating or sleeping.

a: My cat is eating.
b: My cat is sleeping.
c: I'm trying to type.

$$\neg c \rightarrow (a \oplus b)$$

Task 5 – Tea Time

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

a) Define propositional variables and translate the sentence into an expression in logical notation.

p: I am drinking tea.
r: I am eating a cookie.

$$(p \to r) \lor (r \to p)$$

b) Fill out a truth table for your expression.

p	r	$(p \to r)$	$(r \to p)$	$(p \to r) \lor (r \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Task 6 – Truth Tables

Write a truth table for each of the following:

a) $(r \oplus q) \lor (r \oplus \neg q)$

r	q	$r \oplus q$	$r \oplus \neg q$	$(r \oplus q) \lor (r \oplus \neg q)$
T	T	F	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

b) $(r \lor q) \to (r \oplus q)$

r	q	$r \lor q$	$r \oplus q$	$(r \lor q) \to (r \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

c) $p \leftrightarrow \neg p$

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

Task 7 – Interview Question

The following is an old interview question:

There are three boxes, one contains only apples, one contains only oranges, and one contains both apples and oranges. The boxes have been incorrectly labeled such that no label identifies the actual contents of its box. Opening just one box, and without looking in the box, you take out one piece of fruit. By looking at the fruit, how can you immediately label all of the boxes correctly?

a) Create a table showing all the logical possibilities for what could be in each of the boxes. Then indicate which possibilities are consistent with the description in the problem. To simplify the table, you need only list those possibilities where each fruit combination (apples, oranges, or both) appears in exactly one box.

Apples	Oranges	Both	claim
apples	oranges	both	F
apples	both	oranges	F
oranges	apples	both	F
oranges	both	apples	T
both	apples	oranges	T
both	oranges	apples	F

b) If you take a fruit from the box labelled "Apples", will you always know what is in the other boxes? Why or why not?

No. If you happen to get an apple, then you know they must be arranged as in the fifth row, not the fourth row. But if you pick an orange, that is consistent with both the fourth and fifth rows, so you cannot tell which situation you are in.

c) How do you solve the problem?

Pick a fruit from the box labelled "Both". If it is an apple, we are in the fourth row, and if it is an orange, we are in the fifth row.

Task 8 – Equivalences

Prove that each of the following pairs of propositional formulas are equivalent using the specified method(s).

a) $\neg p \to (s \to r)$ vs. $s \to (p \vee r)$ using **(i)** truth tables and **(ii)** propositional equivalences. You can do the equivalence proof on Cozy at https://bit.ly/cse311-23sp-section01-8aii.

(i)

p	r	s	$\neg p$	$(s \to r)$	$(p \lor r)$	$\neg p \to (s \to r)$	$s \to (p \lor r)$
T	T	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	F	F	F
F	F	F	T	T	F	T	T

(ii)

$\neg p \to (s \to r)$	$\equiv$	$\neg\neg p \lor (s \to r)$	Law of Implication
	$\equiv$	$p \lor (s \to r)$	Double Negation
	$\equiv$	$p \lor (\neg s \lor r)$	Law of Implication
	$\equiv$	$(p \lor \neg s) \lor r$	Associativity
	$\equiv$	$(\neg s \lor p) \lor r$	Commutativity
	$\equiv$	$\neg s \lor (p \lor r)$	Associativity
	$\equiv$	$s \to (p \lor r)$	Law of Implication

b) $p \leftrightarrow \neg p$ vs. F (Hint: recall the Biconditional rule $p \leftrightarrow r \equiv (p \to r) \land (r \to p)$) using propositional equivalences. You can do the equivalence proof on Cozy at https://bit.ly/cse311-23sp-section01-8bv2.

$p \leftrightarrow \neg p$	$\equiv$	$(p \to \neg p) \land (\neg p \to p)$	Biconditional
	$\equiv$	$(\neg p \lor \neg p) \land (\neg\neg p \lor p)$	Law of Implication
	$\equiv$	$(\neg p \lor \neg p) \land (p \lor p)$	Double Negation
	$\equiv$	$\neg p \land p$	Idempotence
	$\equiv$	F	Negation

Task 9 – Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent using a truth table and specifying an input they differ on.

a) $p \to r$ vs. $r \to p$

p	r	$p \to r$	$r \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

When $p = $ T and $r = $ F, then $p \to r \equiv$ F, but $r \to p \equiv$ T.

b) $a \to (b \wedge c)$ vs. $(a \to b) \wedge c$

a	b	c	$b \wedge c$	$a \to b$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

When $a =$ F and $c =$ F, then $a \to (b \wedge c) \equiv$ T (by vacuous truth), but $(a \to b) \wedge c \equiv$ F (because c is false).