Quiz Section 1: Propositional Logic Translation

Review

Task 1 – Warm-Up

Translate the English sentences below into symbolic logic.

- a) If I am lifting weights this afternoon, then I do a warm-up exercise.
- b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Task 2 – If I can translate, then...

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

- a) whenever I walk my dog, I make new friends.
- **b)** I will drink coffee, if Starbucks is open or my coffeemaker works.
- c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.
- d) I can go home only if I have finished my homework.
- e) Having an internet connection is necessary to log onto zoom.

Task 3 – I can rewrite these formulas in English, only if...

Given propositions and a logical formula, write **two** potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.

a) p: The sun is out

r: We have class outside

$$p \rightarrow r$$

b) *a*: the book has been out for a week.

c: I have finished reading the book.

$$(a \land b) \rightarrow c$$

- c) p: I have read the manual
 - r: I operate the machine

 $r \rightarrow p$

b: I don't have homework.

Task 4 – Translation

For each of the following, define propositional variables and translate the sentences into logical notation.

- a) I will remember to send you the address only if you send me an e-mail message.
- b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
- c) Unless I am trying to type something, my cat is either eating or sleeping.

Task 5 – Tea Time

Consider the following sentence:

- If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.
- a) Define propositional variables and translate the sentence into an expression in logical notation.
- b) Fill out a truth table for your expression.

Task 6 – Truth Tables

Write a truth table for each of the following:

a)
$$(r \oplus q) \lor (r \oplus \neg q)$$

b)
$$(r \lor q) \to (r \oplus q)$$

c) $p \leftrightarrow \neg p$

Task 7 – Interview Question

The following is an old interview question:

There are three boxes, one contains only apples, one contains only oranges, and one contains both apples and oranges. The boxes have been incorrectly labeled such that no label identifies the actual contents of its box. Opening just one box, and without looking in the box, you take out one piece of fruit. By looking at the fruit, how can you immediately label all of the boxes correctly?

- a) Create a table showing all the logical possibilities for what could be in each of the boxes. Then indicate which possibilities are consistent with the description in the problem. To simplify the table, you need only list those possibilities where each fruit combination (apples, oranges, or both) appears in exactly one box.
- b) If you take a fruit from the box labelled "Apples", will you always know what is in the other boxes? Why or why not?

c) How do you solve the problem?

Task 8 – Equivalences

Prove that each of the following pairs of propositional formulas are equivalent using the specified method(s).

- a) $\neg p \rightarrow (s \rightarrow r)$ vs. $s \rightarrow (p \lor r)$ using (i) truth tables and (ii) propositional equivalences. You can do the equivalence proof on Cozy at https://bit.ly/cse311-23sp-section01-8aii.
- **b)** $p \leftrightarrow \neg p$ vs. F (Hint: recall the Biconditional rule $p \leftrightarrow r \equiv (p \rightarrow r) \land (r \rightarrow p)$) using propositional equivalences. You can do the equivalence proof on Cozy at https://bit.ly/cse311-23sp-section01-8bv2.

Task 9 – Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent using a truth table and specifying an input they differ on.

a)
$$p \rightarrow r$$
 vs. $r \rightarrow p$

b) $a \to (b \land c)$ vs. $(a \to b) \land c$