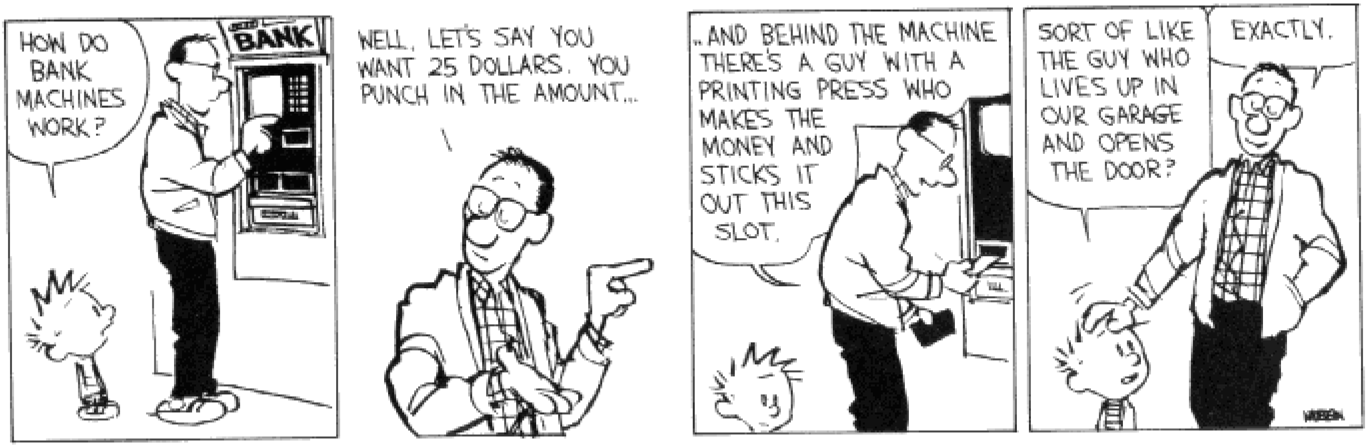
CSE 311: Foundations of Computing

Lecture 22: Finite State Machines


HOW DO BANK MACHINES WORK?
BANK
WELL, LET'S SAY YOU WANT 25 DOLLARS. YOU PUNCH IN THE AMOUNT...
..AND BEHIND THE MACHINE THERE'S A GUY WITH A PRINTING PRESS WHO MAKES THE MONEY AND STICKS IT OUT THIS SLOT.
SORT OF LIKE THE GUY WHO LIVES UP IN OUR GARAGE AND OPENS THE DOOR?
EXACTLY.
WATTERSON

About the Midterm

We will release midterm grades at the end of the day today.

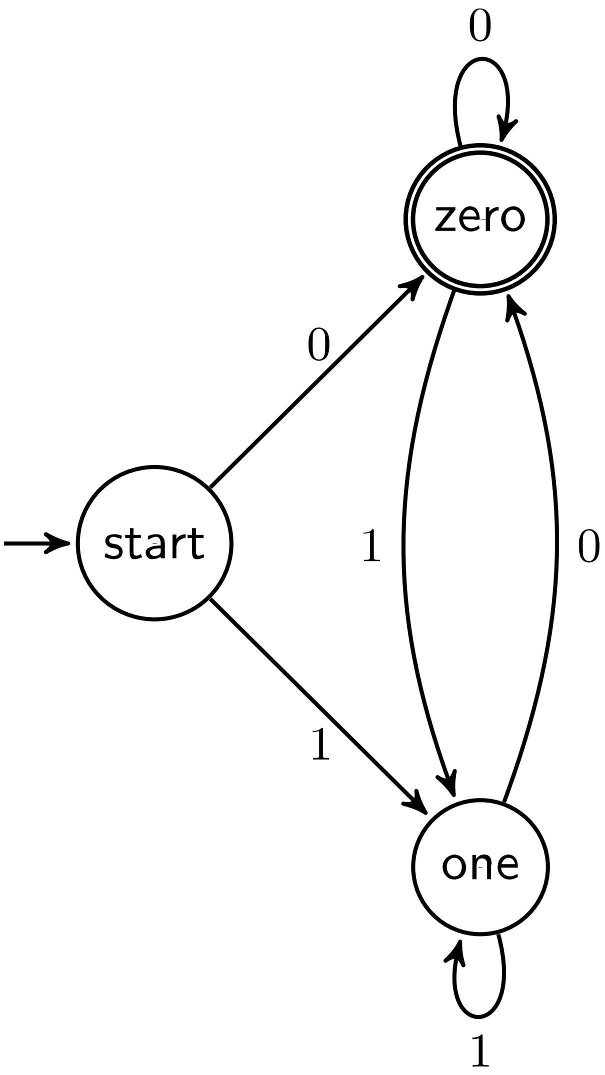
Preliminary information:

Median: 81 Average: 75 Standard Deviation: 19

Grade Distribution:

90+	25%
80’s	29%
70’s	14%
60’s	11%
50’s	9%
40’s	7%
<40	5%

Last class: Strings this machine says are OK?

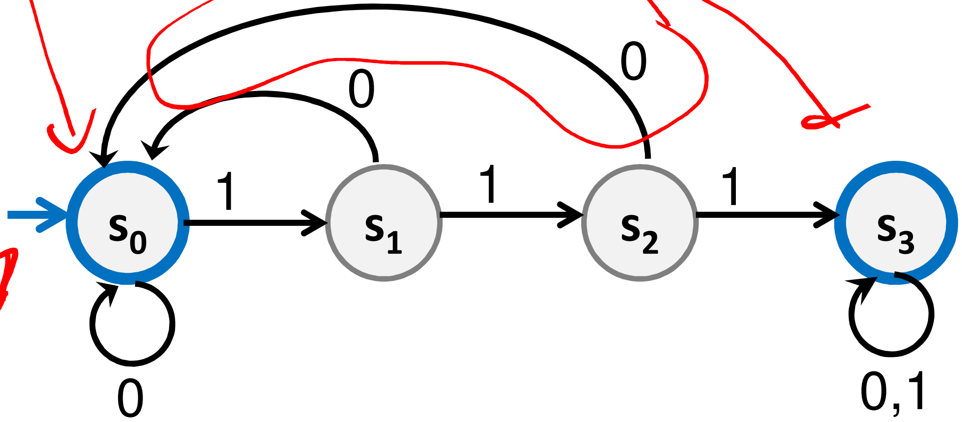


The set of all binary strings that end in 0

Finite State Machines

- States
- Transitions on input symbols
- Start state and final states
- The "language recognized" by the machine is the set of strings that reach a final state from the start

Old State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3

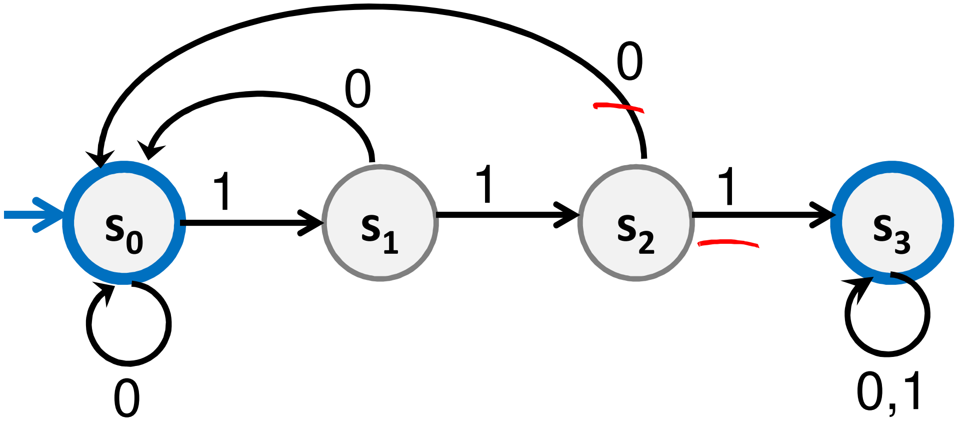


Finite State Machines

- Each machine designed for strings over some fixed alphabet Σ.

- Must have a transition defined from each state for *every* symbol in Σ.

Old State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3



What language does this machine recognize?

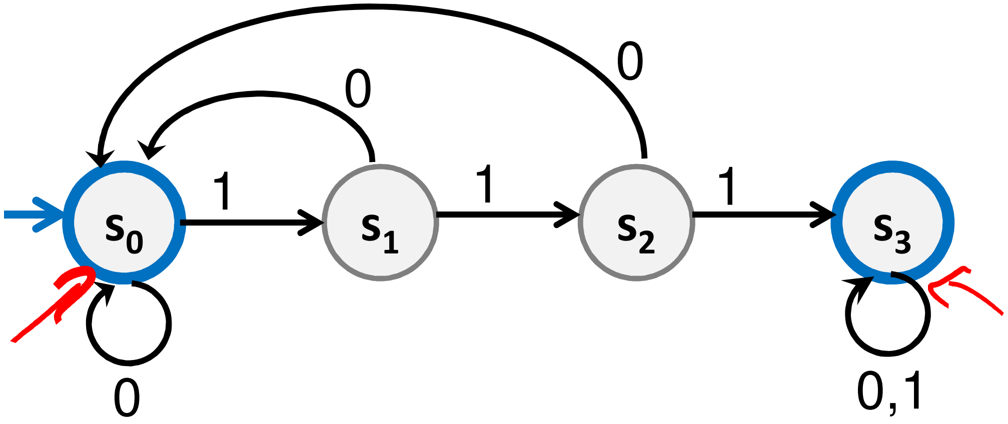
Binary strings

Contain 111

or ending in 0

or ε

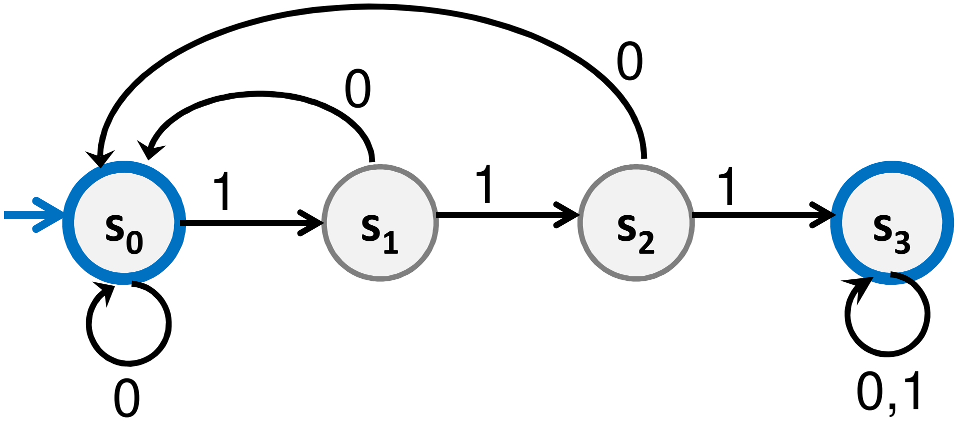
Old State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3



What language does this machine recognize?

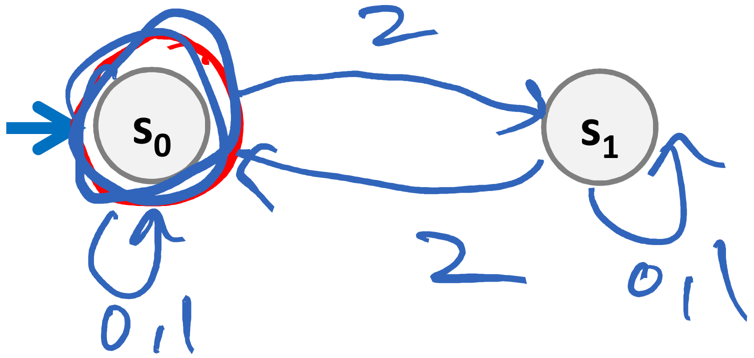
The set of all binary strings that contain 111 or don’t end in 1

Old State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3



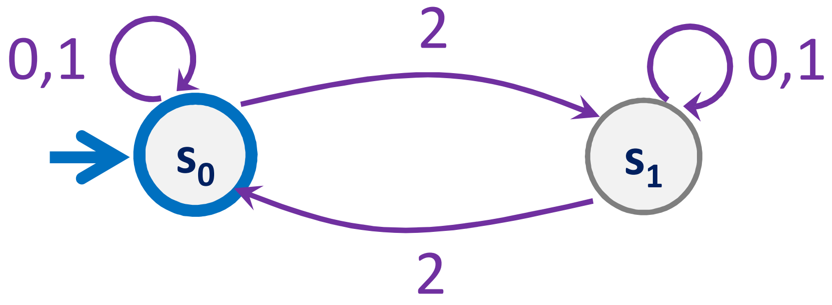
Strings over {0, 1, 2}

M_1: Strings with an even number of 2's



Strings over {0, 1, 2}

M_1: Strings with an even number of 2's



State Machine Design Recipe

Given a language, how do you design a state machine for it?

Create states to remember enough

(about the portion of the input string that it has already seen)

to correctly answer “accept/reject” on the whole string after seeing the rest.

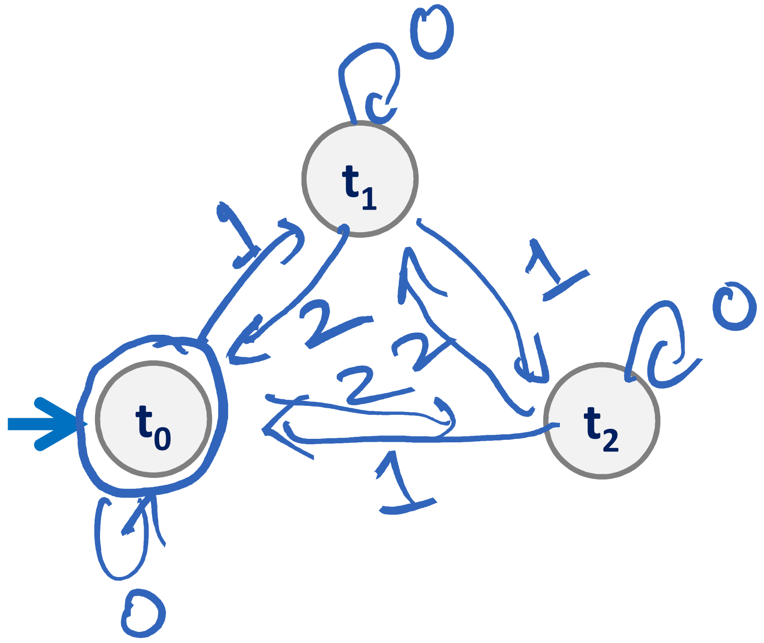
Add labeled edges to show how the memory (state) should be updated for each new symbol.

Strings over {0, 1, 2}

M_2: Strings where the sum of digits mod 3 is 0

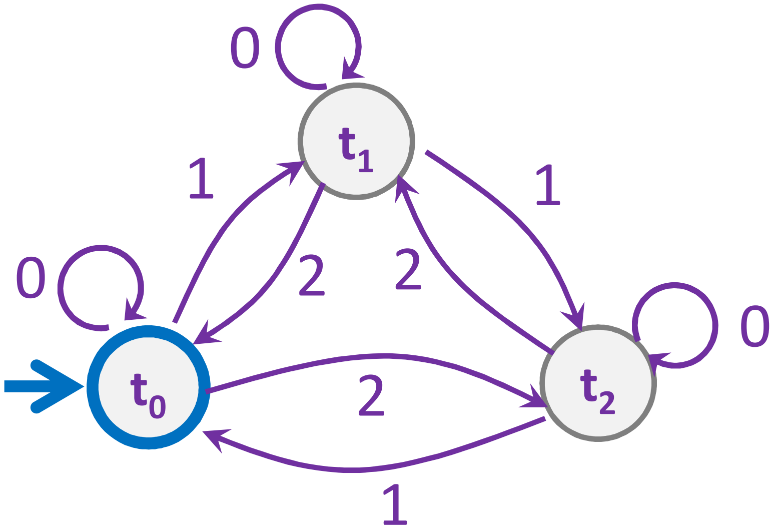
Strings over {0, 1, 2}

M_2: Strings where the sum of digits mod 3 is 0

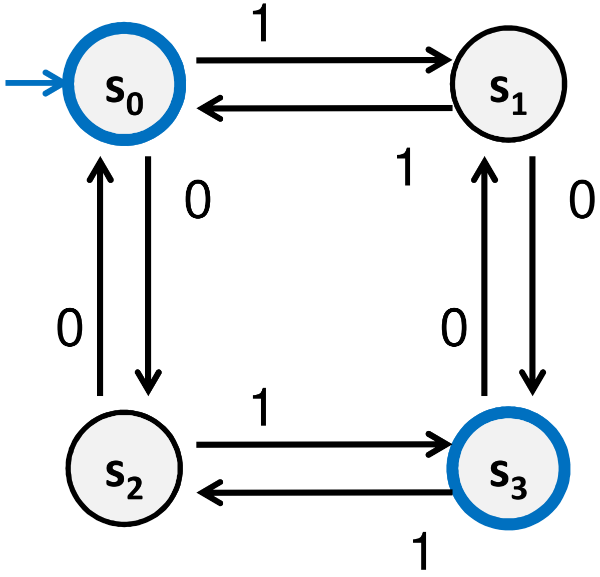


Strings over {0, 1, 2}

M_2: Strings where the sum of digits mod 3 is 0



What language does this machine recognize?

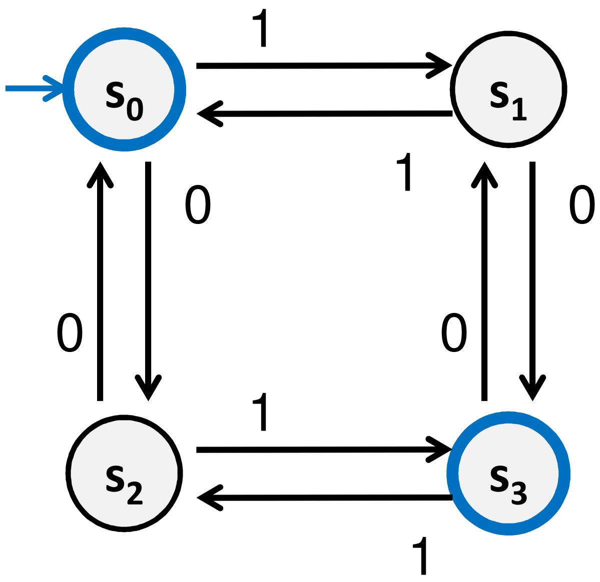


Good
ε
10
00
01
11

Bad
1
0
111
110
101
000
.
.
.

Binary string with an even ~~# of bits~~ length

What language does this machine recognize?



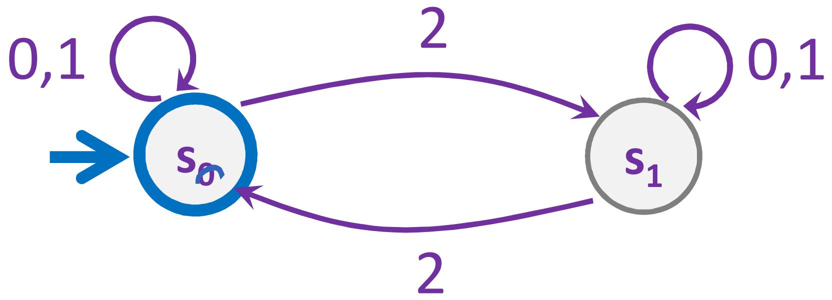
The set of all binary strings with # of 1's ≡ # of 0's (mod 2) (both are even or both are odd).

Can you think of a simpler description?

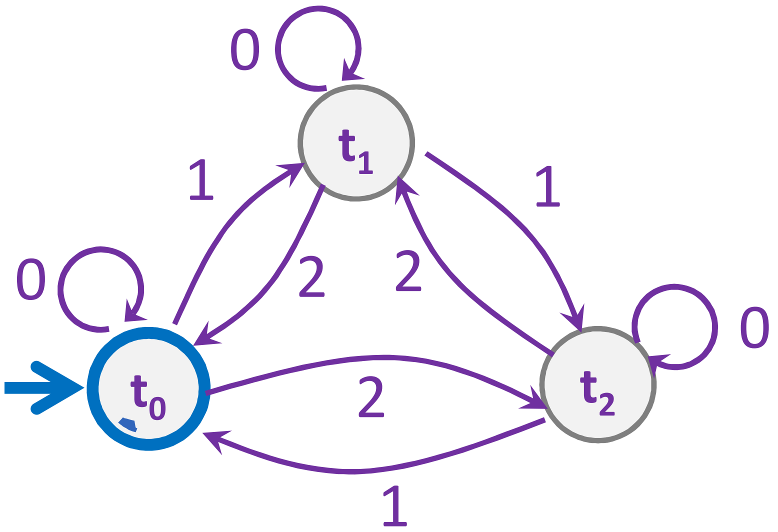
We got this one

Strings over {0, 1, 2}

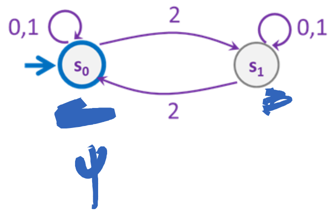
M_1: Strings with an even number of 2's

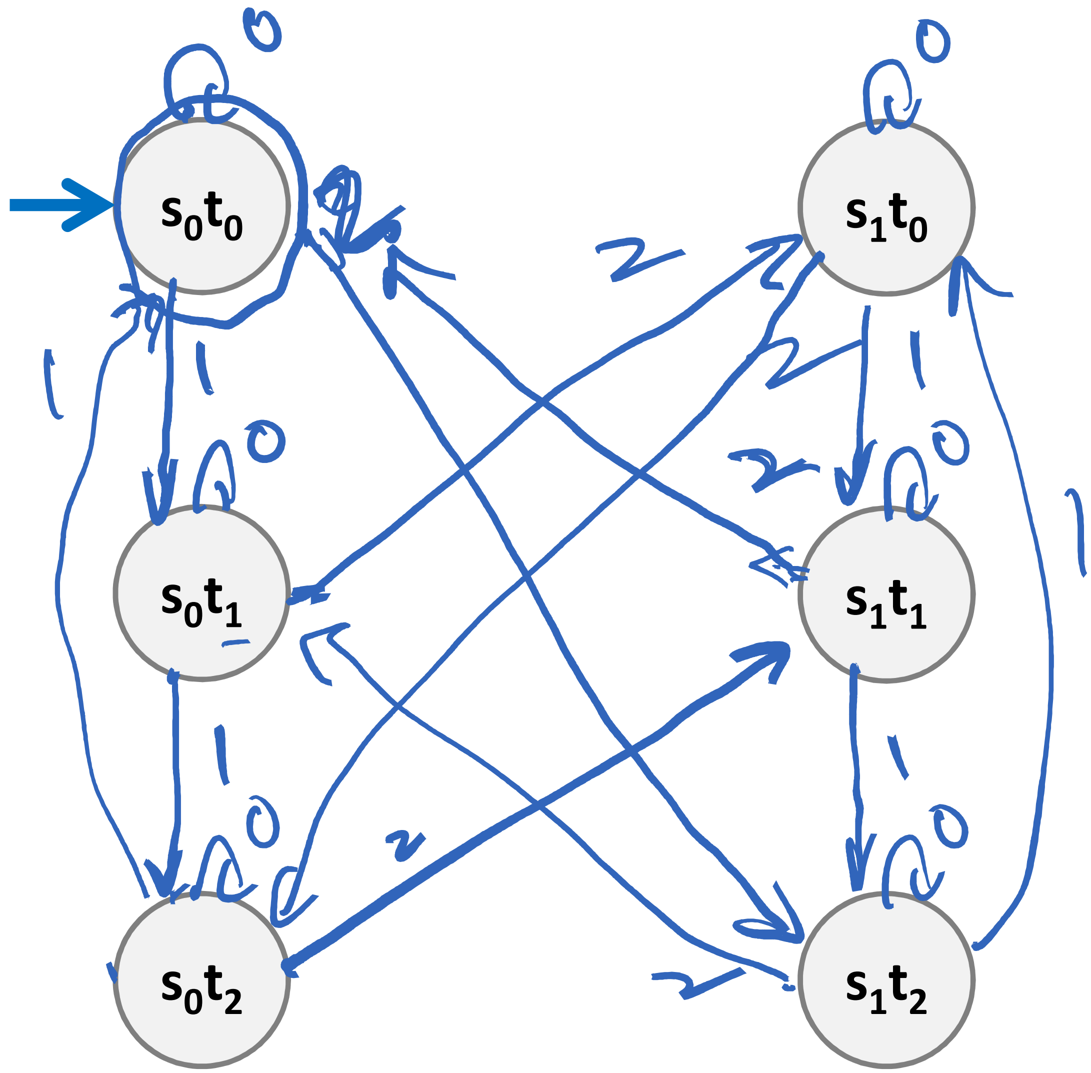


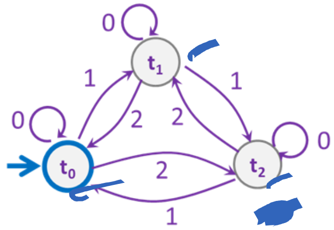
M_2: Strings where the sum of digits mod 3 is 0



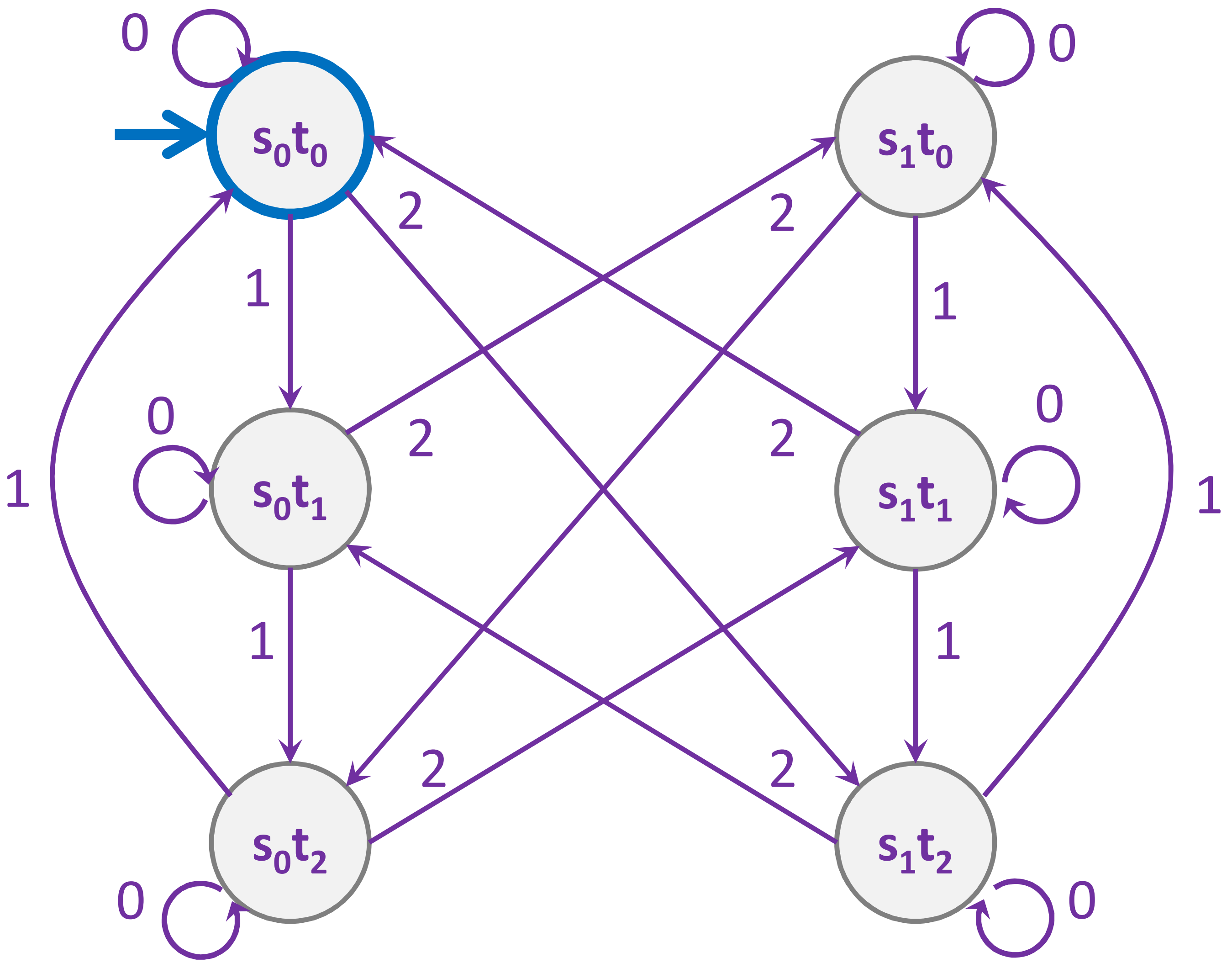
Strings over {0,1,2} w/ even number of 2’s and mod 3 sum 0



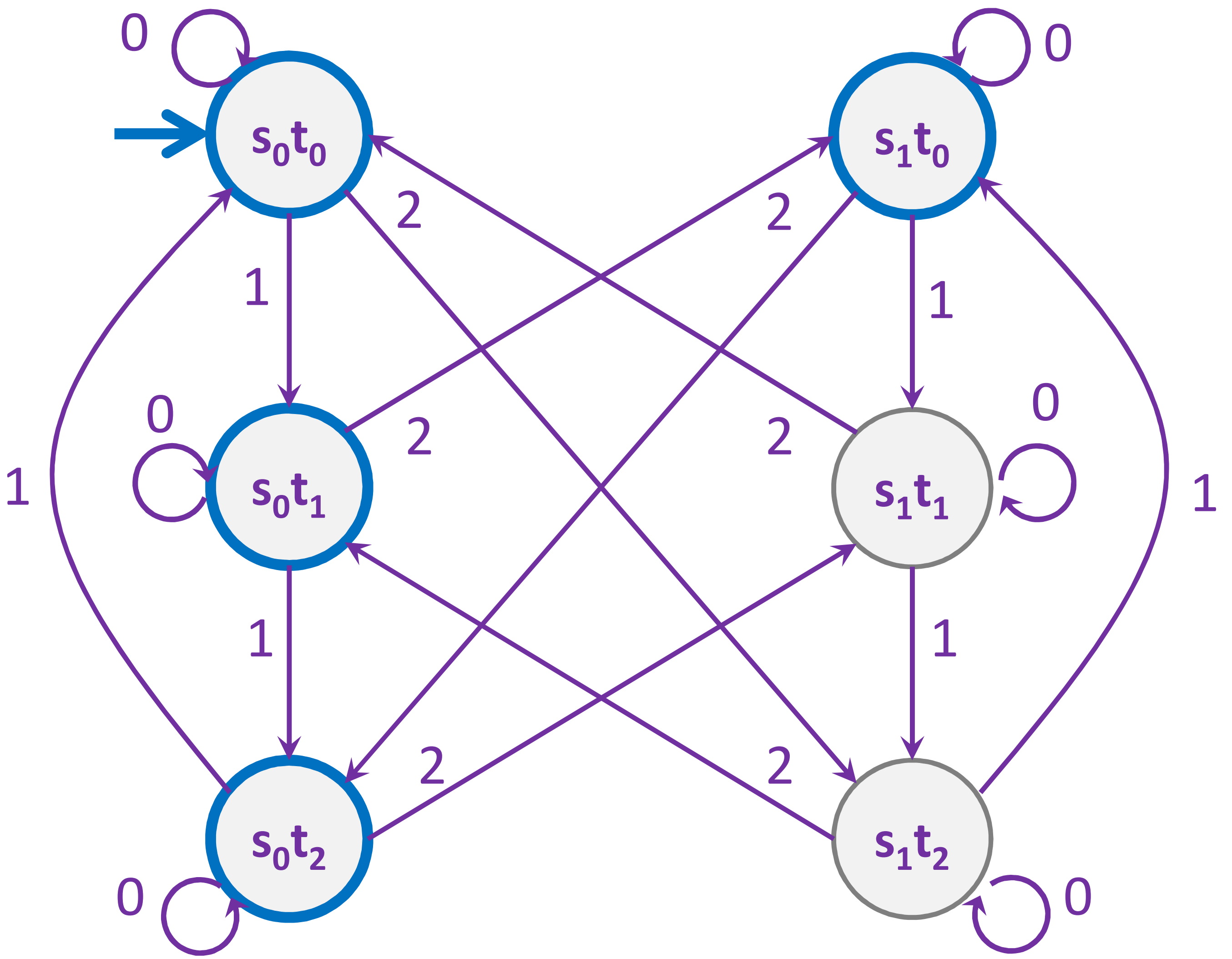




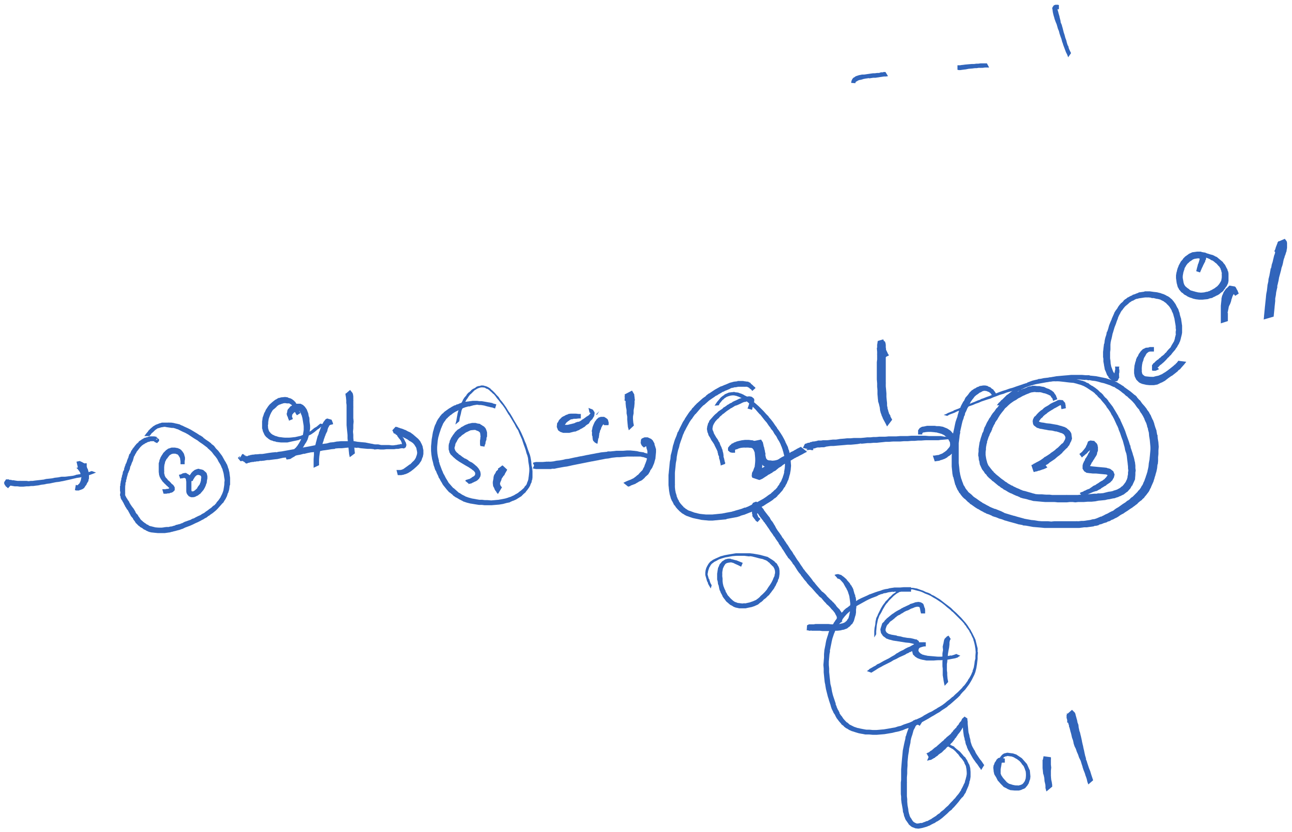
Strings over {0,1,2} w/ even number of 2’s and mod 3 sum 0



Strings over {0,1,2} w/ even number of 2's OR mod 3 sum 0



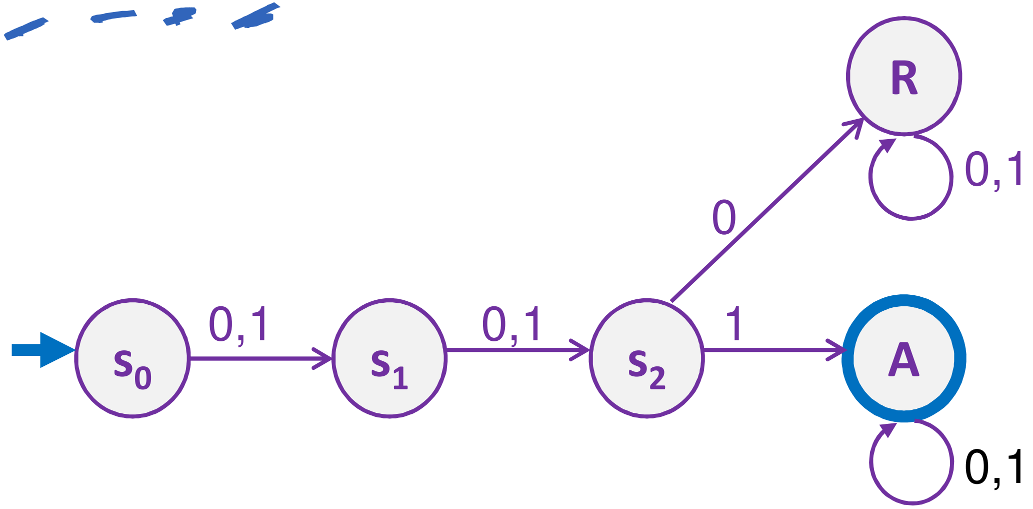
The set of binary strings with a 1 in the 3rd position from the start



The set of binary strings with a 1 in the 3rd position from the start

1001

_ _ _ _

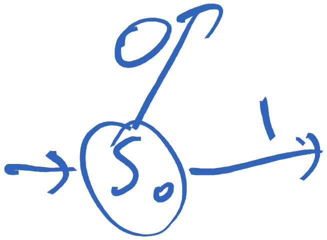


The set of binary strings with a 1 in the 3rd position from the end

10 0 0 ✗

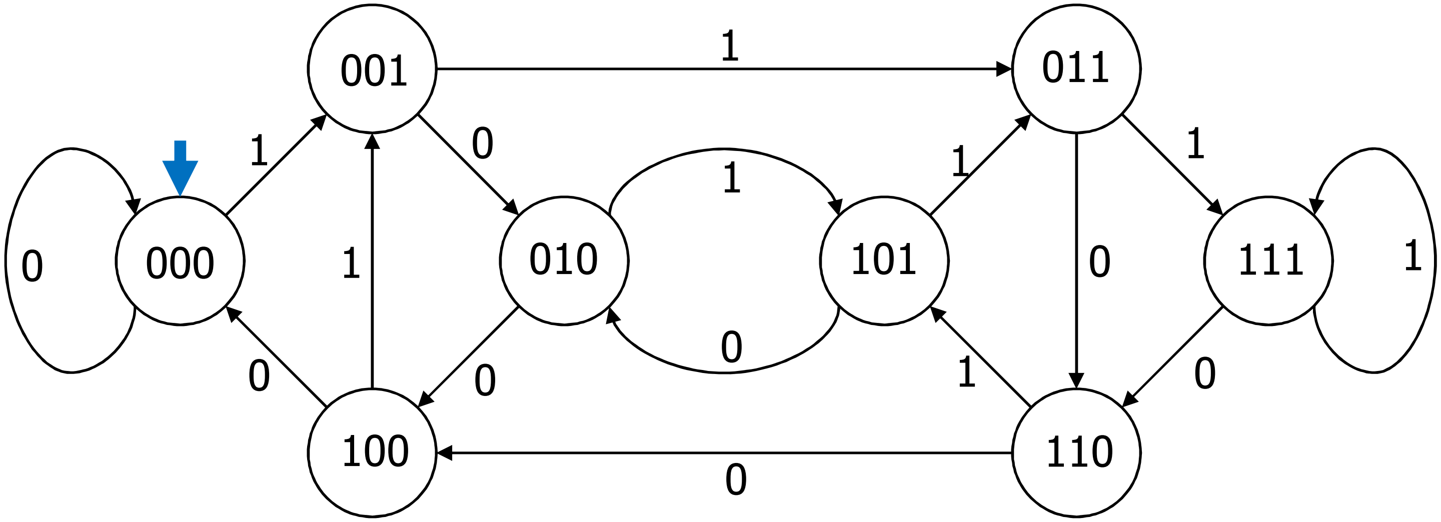
1000

1100 ✓

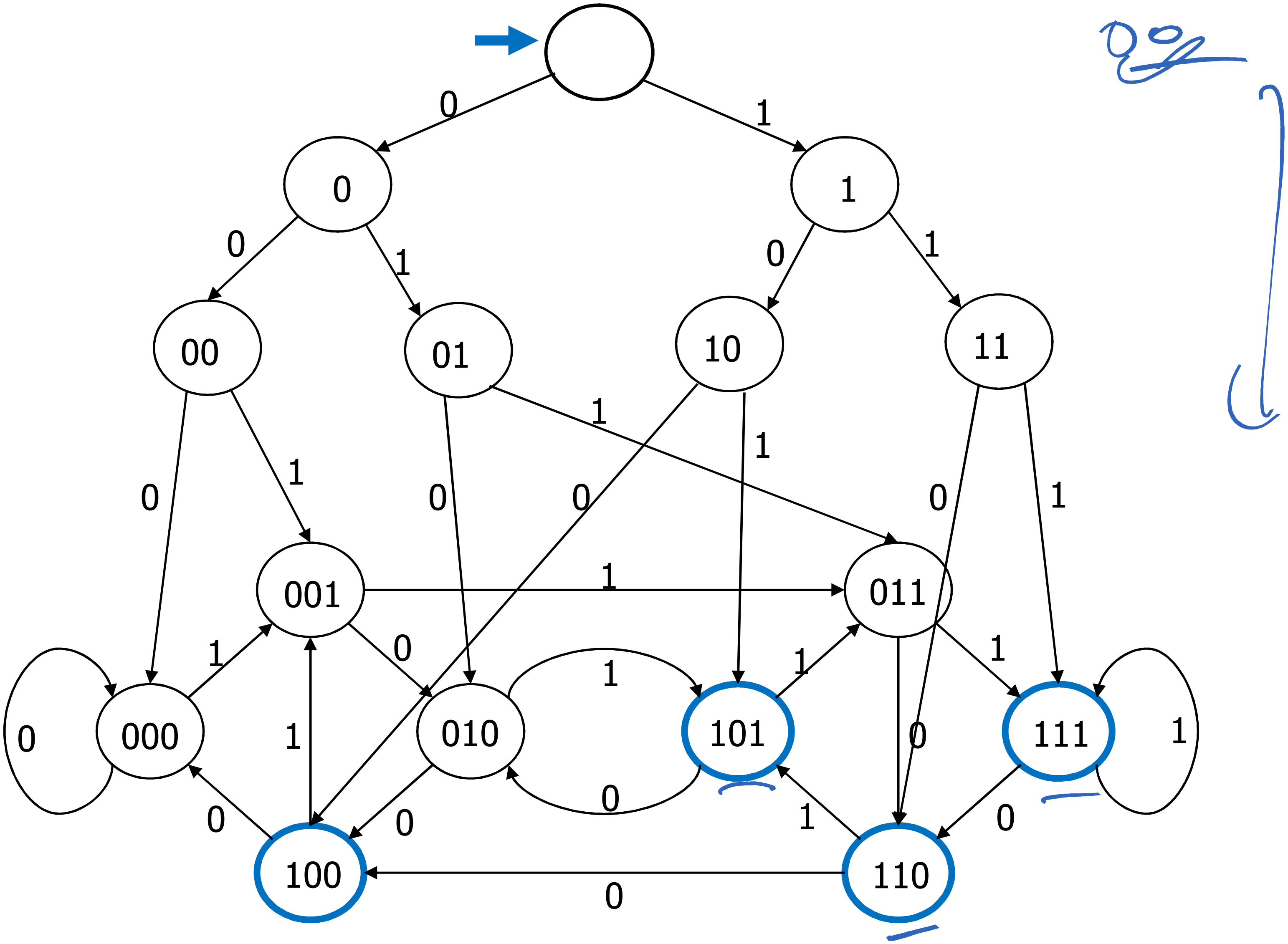


100

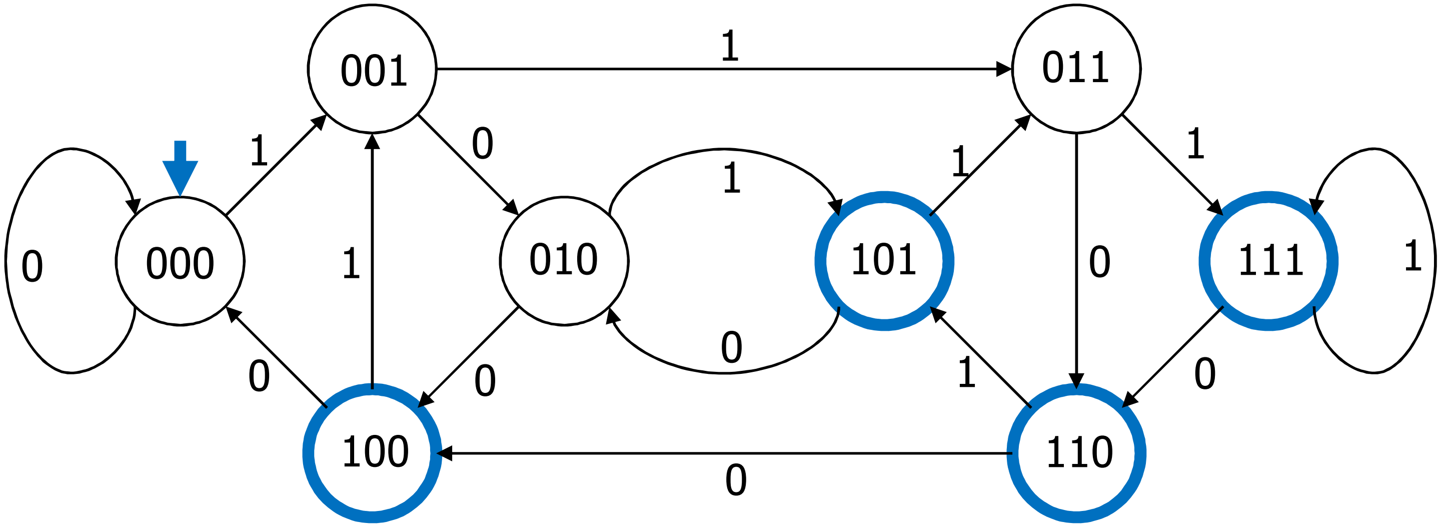
3 bit shift register “Remember the last three bits”



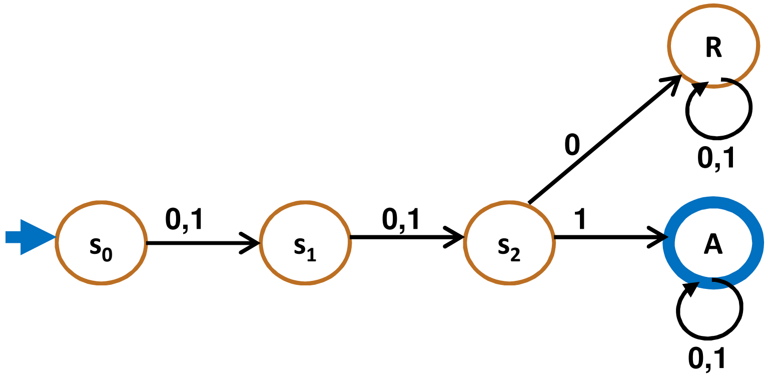
The set of binary strings with a 1 in the 3rd position from the end

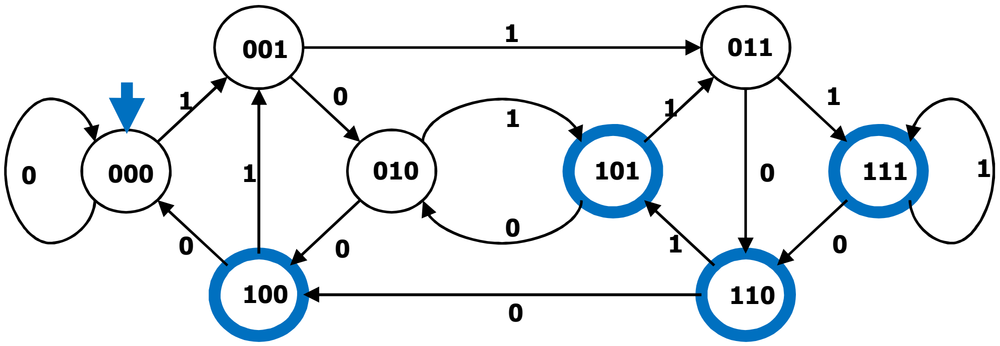


The set of binary strings with a 1 in the 3rd position from the end



The beginning versus the end





Adding Output to Finite State Machines

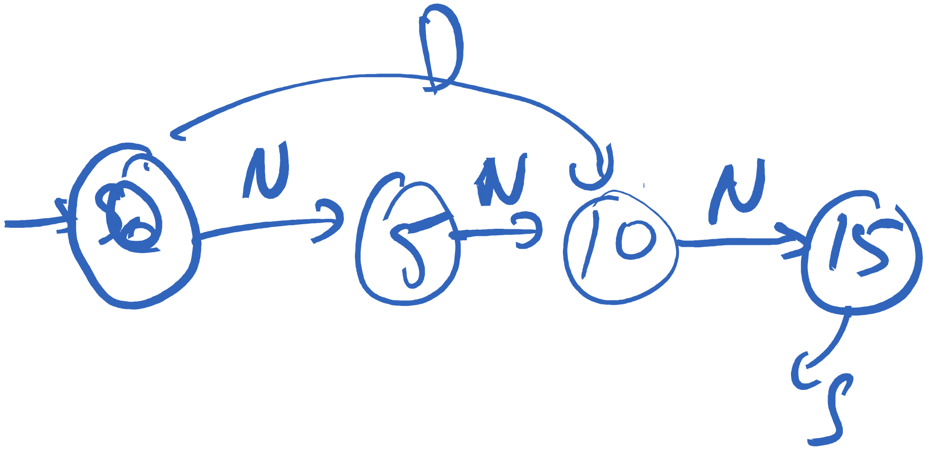
- So far, we have considered finite state machines that just accept/reject strings
  - called “Deterministic Finite Automata” or DFAs

- Now we consider finite state machines *with output*
  - These are the kinds used as controllers

Vending Machine

Enter 15 cents in dimes or nickels

Press S or B for a candy bar


D
N
N
N
0
5
10
15
S