Problem Set 7

Due: Wednesday, May 24, by 11:59pm

Instructions

Solutions submission. You must submit your solution via Gradescope. In particular:

- Submit a single PDF file in Gradescope containing the written solution to all the regular tasks 1-4 and your "documentation" sentences for task 5 (see below for an explanation).
- Submit your solution to task 5 online to grin.cs as described in the task itself.
- The extra credit is submitted separately in Gradescope

Task 1 – Glitz and Grammar [18 pts]

Construct a context-free grammar that generates each of the following sets of strings.

"Document" all the non-start variables in your grammar with an English description of the set of strings it generates. (You do not need to document the start variable because it is documented by the problem statement.)

a) Binary strings matching the regular expression "$1(0 \cup 111)^* \cup 000$".

Hint: You can use the technique described in lecture to convert this RE to a CFG.

b) All strings of the form $x\#y$, with $x, y \in \{0,1\}^*$ and x a subsequence of y^R.

(Here y^R means the reverse of y. Also, a string w is a subsequence of another string z if you can delete some characters from z to arrive at w.)

c) All binary strings in the set $\{0^m 1^n 0^{2n+m} : m, n \geqslant 0\}$.

Task 2 – 101 Relations [20 pts]

For each of the relations below, determine whether or not it has each of the properties of reflexivity, symmetry, antisymmetry, and/or transitivity. If a relation has a property, simply say so without any further explanation. If a relation does not have a property, state a counterexample, but do not explain your counterexample further.

a) Define $S \subseteq \mathbb{Z} \times \mathbb{Z}$ by $(a,b) \in S$ iff $|a-b| \leqslant 3$ (where $|x|$ denotes the absolute value of x, i.e., x if x is nonnegative, and $-x$ if x is negative).

b) Define $T \subseteq \mathbb{Z} \times \mathbb{Z}$ by $(a,b) \in T$ iff $a \neq b$.

c) Let $A = \{n \in \mathbb{N} : n > 0\}$ be the set of positive natural numbers. Define $U \subseteq A \times A$ by $(a,b) \in U$ iff $a \mid b$, i.e., a divides b.

d) Let $B = \mathcal{P}(\mathbb{Z})$. Define $V \subseteq B \times B$ by $(X, Y) \in V$ iff $X \cap [10] \subseteq Y \cap [10]$. (Remember that $[n] = \{1, \ldots, n\}$.)

e) Let $A = \{n \in \mathbb{N} : n > 0\}$ be the set of positive natural numbers. Define $W \subseteq A \times A$ by $(a, b) \in W$ iff $a \mid 2$ and $b \mid 3$ and ($a = 1$ or $b = 1$).

Hint: Careful! The definition of W says $a \mid 2$, which is different from (the more common) $2 \mid a$.

Task 3 – Get a Prove On [18 pts]

Let A be a set and let R and S be relations on A. Suppose that R and S are transitive.

When asked to state a counterexample below, you need to give a specific set A and specific transitive relations R and S and then describe why they constitute a counterexample to the claim.

a) Is $R \cap S$ transitive? If so, prove it. If not, state a counterexample (and explain why it is a counterexample).

b) Is $R \cup S$ transitive? If so, prove it. If not, state a counterexample (and explain why it is a counterexample).

Task 4 – Cat Goes "Meow"; Dog Goes "Proof" [14 pts]

Let A be a set and let R and S be relations on A. Suppose that R is symmetric and S is symmetric. Further suppose that $R \circ S = S \circ R$. Prove that $R \circ S$ is symmetric.

Task 5 – Few and Far Machine [30 pts]

For each of the following, create a *DFA* that recognizes exactly the language given.
For all states in your DFA, include "documentation" for them by describing, in English, the set of strings that *end* in that state.

a) Binary strings where every occurrence of a 0 is immediately followed by a 1.

b) Binary strings that start with 0 and have even length.

c) Binary strings with an even number of 0s.

d) Binary strings with at least two 0s.

e) Binary strings with at least two 0s **or** at least two 1s.

Submit and check your answers to this question here: `https://grin.cs.washington.edu/` Think carefully about your answer to make sure it is correct before submitting. You have only 3 chances to submit a correct answer. You must *also* submit your "documentation" on Gradescope along with your written solutions to other tasks. Please include a screenshot/sketch of your DFA with your documentation so that we can easily grade it.

Task 6 – Extra Credit: With a Grammar, the Whole World is a Nail

Consider the following context-free grammar.

$$
\begin{array}{ll}
\langle\textbf{Stmt}\rangle & \rightarrow \langle\textbf{Assign}\rangle \mid \langle\textbf{IfThen}\rangle \mid \langle\textbf{IfThenElse}\rangle \mid \langle\textbf{BeginEnd}\rangle \\
\langle\textbf{IfThen}\rangle & \rightarrow \text{if condition then } \langle\textbf{Stmt}\rangle \\
\langle\textbf{IfThenElse}\rangle & \rightarrow \text{if condition then } \langle\textbf{Stmt}\rangle \text{ else } \langle\textbf{Stmt}\rangle \\
\langle\textbf{BeginEnd}\rangle & \rightarrow \text{begin } \langle\textbf{StmtList}\rangle \text{ end} \\
\langle\textbf{StmtList}\rangle & \rightarrow \langle\textbf{StmtList}\rangle\langle\textbf{Stmt}\rangle \mid \langle\textbf{Stmt}\rangle \\
\langle\textbf{Assign}\rangle & \rightarrow \texttt{a := 1}
\end{array}
$$

This is a natural-looking grammar for part of a programming language, but unfortunately the grammar is "ambiguous" in the sense that it can be parsed in different ways (that have distinct meanings).

a) Show an example of a string in the language that has two different parse trees that are meaningfully different (i.e., they represent programs that would behave differently when executed).

b) Give **two different grammars** for this language that are both unambiguous but produce different parse trees from each other.