Propositional Logic
Administrivia & Introductions
Homework

- Submissions
  - LaTeX (highly encouraged)
    - overleaf.com
    - template and LaTeX guide posted on course website!
  - Word Editor that supports mathematical equations
  - Handwritten neatly and scanned

- All homeworks will be turned in via Gradescope

- Homeworks typically due on Fridays at 10pm

- You have 6 late days **total** to use throughout the quarter
  - Anything beyond that will result in a deduction on further late assignments

- Only 3 late days max can be used per assignment
Announcements & Reminders

● Sections are Graded
  ○ You will be graded on section participation, so please try to come 😊

● Section Materials
  ○ Handouts will be provided in at each section
  ○ Worksheets and sample solutions will be available on the course calendar later this evening

● HW1
  ○ Due Friday 10/6 @ 10pm
Icebreaker

- Small groups of 4-6ish
- Please share with your group
  - Your name
  - Number of years in department/ at UW
  - What was something fun you did over Summer break?
  - What are you concerned about for 311 / what are you excited about?
- Then, share how you like to eat your potatoes (baked, fried, chips, etc.)
- We’ll go around and see what style of potato is most popular!
Propositions & Implications
Quick Concept Review

- **Propositions** are statements with a boolean truth value!
  - “The AQI of Seattle is 50” is a proposition. We know it’s either true or false.
  - “The AQI of Seattle?” is not. Suddenly it could be hundreds of values.
  - In formal logic, we like to assign a proposition into a variable for later use.

- **Logical connectives** connect propositions to form new propositions!

  \[
  \neg p \\
  p \land q \\
  p \lor q \\
  p \rightarrow q \\
  p \leftrightarrow q
  \]
Truth Tables

Gives us a simple way to describe how logical connectives operate

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Implications

Some common formulations:

- $p$ implies $q$
- Whenever $p$ is true $q$ must be true
- If $p$ then $q$
- $q$ if $p$
- $p$ is sufficient for $q$
- $p$ only if $q$
- $q$ is necessary for $p$

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Problem 1 – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

(a) If I am lifting weights this afternoon, then I do a warm-up exercise.

(b) If I am cold and going to bed or I am two-years old, then I carry a blanket.
Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.
Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Step 1

\begin{align*}
p & : \text{I am lifting weights this afternoon} \\
q & : \text{I do a warm-up exercise}
\end{align*}
Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Step 1
\( p \): I am lifting weights this afternoon
\( q \): I do a warm-up exercise

Step 2
If \( p \) then \( q \)
Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

**Step 1**

\[ p: \text{I am lifting weights this afternoon} \]
\[ q: \text{I do a warm-up exercise} \]

**Step 2**

If \( p \) then \( q \)

**Step 3**

\[ p \rightarrow q \]
Problem 1b
Problem 1b – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Work on this problem with the people around you, and then we’ll go over it together!
Problem 1b – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.
Problem 1b – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Step 1

\[ p: \text{I am cold} \]
\[ q: \text{I am going to bed} \]
\[ r: \text{I am two-years old} \]
\[ s: \text{I carry a blanket} \]

**NOTE:** you need a subject for each proposition. “Going to bed” is not a proper proposition, you need to add the “I am” to make it a valid sentence, and thus a valid proposition!!!
Problem 1b – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

**Step 1**

p: I am cold
q: I am going to bed
r: I am two-years old
s: I carry a blanket

**Step 2**

If p and q or r, then s
Problem 1b – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

**Step 1**

\[ p: \text{I am cold} \]
\[ q: \text{I am going to bed} \]
\[ r: \text{I am two-years old} \]
\[ s: \text{I carry a blanket} \]

**Step 2**

If \( p \) and \( q \) or \( r \), then \( s \)

**Step 3**

\[ [(p \land q) \lor r] \rightarrow s \]
Problem 2
Problem 2 – If I can translate, then…

a) Whenever I walk my dog, I make new friends.

b) I will drink coffee, if Starbucks is open or my coffeemaker works.

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

d) I can go home only if I have finished my homework.

e) Having an internet connection is necessary to log onto zoom.

f) I am a student because I attend university.

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

Work on parts (a), (c), and (f) with the people around you, and then we’ll go over it together!
Problem 2 – If I can translate, then...

a) Whenever I walk my dog, I make new friends.

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

a) Whenever I walk my dog, I make new friends.

Step 1

$p$: I walk my dog
$q$: I make new friends
Problem 2 – If I can translate, then...

a) Whenever I walk my dog, I make new friends.

Step 1
$p$: I walk my dog
$q$: I make new friends

Step 2
Whenever $p$, $q$
If $p$ then $q$

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

a) Whenever I walk my dog, I make new friends.

Step 1

$p$: I walk my dog
$q$: I make new friends

Step 2

Whenever $p$, $q$
If $p$ then $q$

Step 3

$p \rightarrow q$

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

Step 1
\( p \): One is a U.S. Citizen
\( q \): One is over 18
\( r \): One is eligible to vote

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

**Step 1**

\(p\): One is a U.S. Citizen
\(q\): One is over 18
\(r\): One is eligible to vote

**Step 2**

Being \(p\) and \(q\) is sufficient for \(r\)
If \(p\) and \(q\) then \(r\)

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

Step 1
$p$: One is a U.S. Citizen
$q$: One is over 18
$r$: One is eligible to vote

Step 2
Being $p$ and $q$ is sufficient for $r$
If $p$ and $q$ then $r$

Step 3
$(p \land q) \rightarrow r$
Problem 2 – If I can translate, then...

f) I am a student because I attend university.

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

f) I am a student because I attend university.

**Step 1**

\( p \): I am a student
\( q \): I attend university

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

f) I am a student because I attend university.

Step 1
𝑝: I am a student
𝑞: I attend university

Step 2
𝑝 because 𝑞
If 𝑞 then 𝑝

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators
Problem 2 – If I can translate, then...

f) I am a student because I attend university.

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

**Step 1**
\( p \): I am a student
\( q \): I attend university

**Step 2**
\( p \) because \( q \)
If \( q \) then \( p \)

**Step 3**
\( q \rightarrow p \)
Problem 5
Problem 5 – Tea Time

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

a) Define propositional variables and translate the sentence into an expression in logical notation.

b) Fill out a truth table for your expression.

Work on this problem with the people around you, and then we’ll go over it together!
Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

a) Define propositional variables and translate the sentence into an expression in logical notation.
Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

a) Define propositional variables and translate the sentence into an expression in logical notation.

\[ p: \text{I am drinking tea} \]
\[ q: \text{I am eating a cookie} \]
Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

a) Define propositional variables and translate the sentence into an expression in logical notation.

\( p: \) I am drinking tea
\( q: \) I am eating a cookie

\( (p \rightarrow q) \lor (q \rightarrow p) \)
Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

b) Fill out a truth table for your expression. \((p \rightarrow q) \lor (q \rightarrow p)\)

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Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

b) Fill out a truth table for your expression. \((p \rightarrow q) \lor (q \rightarrow p)\)

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Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

b) Fill out a truth table for your expression. \((p \rightarrow q) \vee (q \rightarrow p)\)

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Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

b) Fill out a truth table for your expression. \((p \rightarrow q) \lor (q \rightarrow p)\)

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Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

b) Fill out a truth table for your expression. \((p \to q) \lor (q \to p)\)

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If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

b) Fill out a truth table for your expression. \((p \rightarrow q) \lor (q \rightarrow p)\)

\[
\begin{array}{c|c|c|c|c|}
 p & q & p \rightarrow q & q \rightarrow p & (p \rightarrow q) \lor (q \rightarrow p) \\
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\end{array}
\]
Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

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Problem 5 – Tea Time

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

b) Fill out a truth table for your expression. 

\[(p \rightarrow q) \lor (q \rightarrow p)\]

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That’s All, Folks!

Thanks for coming to section this week!
Any questions?