

# Weak Induction Template

1. Define  $P(n)$ . State that your proof is by induction on  $n$ .
2. Base Case: Show  $P(b)$  i.e. show the base case
3. Inductive Hypothesis: Suppose  $P(k)$  for an arbitrary  $k \geq b$ .
4. Inductive Step: Show  $P(k + 1)$  (i.e. get  $P(k) \rightarrow P(k + 1)$ )
5. Conclude by saying  $P(n)$  is true for all  $n \geq b$  by the principle of induction.

# Strong Induction Template (with multiple base cases)

1. Define  $P(n)$ . State that your proof is by induction on  $n$ .
2. Base Cases: Show  $P(b_{min}), P(b_{min+1}) \dots P(b_{max})$  i.e. show the base cases
3. Inductive Hypothesis: Suppose  $P(b_{min}) \wedge P(b_{min} + 1) \wedge \dots \wedge P(k)$  for an arbitrary  $k \geq b_{max}$ . (The smallest value of  $k$  assumes **all** bases cases, but nothing else)
4. Inductive Step: Show  $P(k + 1)$  (i.e. get  $[P(b_{min}) \wedge \dots \wedge P(k)] \rightarrow P(k + 1)$ )
5. Conclude by saying  $P(n)$  is true for all  $n \geq b_{min}$  by the principle of induction.

# Structural Induction Template

1. Define  $P()$  Show that  $P(x)$  holds for all  $x \in S$ . State your proof is by structural induction.

2. Base Case: Show  $P(x)$

[Do that for every base cases  $x$  in  $S$ .]

Let  $y$  be an arbitrary element of  $S$  not covered by the base cases. By the exclusion rule,  $y = \langle \text{recursive rules} \rangle$

3. Inductive Hypothesis: Suppose  $P(x)$

[Do that for every  $x$  listed as in  $S$  in the recursive rules.]

4. Inductive Step: Show  $P()$  holds for  $y$ .

[You will need a separate case/step for every recursive rule.]

5. Therefore  $P(x)$  holds for all  $x \in S$  by the principle of induction.