CSE 311 Midterm Review!
Administrivia
Announcements & Reminders

TONIGHT @ 6-7:30 pm in BAG 131 and 154

● Please **bring an ID** (Husky Card or other ID) to the exam. We’ll be checking those during the exam.

● Remember you’re allowed one piece of paper of handwritten notes. Please read details on the [exams](#) page.

● **Check your email** for room assignments
Set Theory
Problem 4- Section 04

Write an English proof, proving the following set identity

Let the universal set be $U$. Prove $A \cap B' \subseteq A \setminus B$ for any sets $A, B$.

Work on this problem with the people around you.

(1) Translate the claim to predicate
(2) Write out the skeleton
Problem 4- Section 04
Write an English proof, proving the following set identity

Let the universal set be $U$. Prove $A \cap B' \subseteq A \setminus B$ for any sets $A$, $B$.

Work on this problem with the people around you.

(1) Translate the claim to predicate
\[ \forall x (x \in A \cap B' \rightarrow x \in A \setminus B) \]

(2) Write out the skeleton
Remember the Skeleton!

How would we show $A \subseteq B$?

Let $x$ be an arbitrary element of $A$

... 

So $x$ is also in $B$.

Since $x$ was an arbitrary element of $A$, we have that $A \subseteq B$. 

$A \subseteq B \equiv \forall x(x \in A \rightarrow x \in B)$
Let's Write it Out: \( \forall x(x \in A \cap B' \rightarrow x \in A\setminus B) \)
Problem 4 - Section 04

Write an English proof, proving the following set identity

Let the universal set be $U$. Prove $A \cap B' \subseteq A \setminus B$ for any sets $A, B$.

Let $x$ be an arbitrary element and suppose that $x \in A \cap B'$.

So $x \in A \setminus B$.

Since $x$ was arbitrary, we can conclude that $A \cap B' \subseteq A \setminus B$ by definition of subset.
Problem 4- Section 04

Write an English proof, proving the following set identity

Let the universal set be $U$. Prove $A \cap B' \subseteq A \setminus B$ for any sets $A$, $B$.

Let $x$ be an arbitrary element and suppose that $x \in A \cap B'$. By definition of intersection, $x \in A$ and $x \in B'$.

...So $x \in A \setminus B$.

Since $x$ was arbitrary, we can conclude that $A \cap B' \subseteq A \setminus B$ by definition of subset.
Let the universal set be \( U \). Prove \( A \cap B' \subseteq A \setminus B \) for any sets \( A, B \).

Let \( x \) be an arbitrary element and suppose that \( x \in A \cap B' \). By definition of intersection, \( x \in A \) and \( x \in B' \). So by definition of complement, \( x \notin B \).

So \( x \in A \setminus B \).

Since \( x \) was arbitrary, we can conclude that \( A \cap B' \subseteq A \setminus B \) by definition of subset.
Problem 4- Section 04
Write an English proof, proving the following set identity

Let the universal set be \( U \). Prove \( A \cap B' \subseteq A \setminus B \) for any sets \( A, B \).

Let \( x \) be an arbitrary element and suppose that \( x \in A \cap B' \).
By definition of intersection, \( x \in A \) and \( x \in B' \).
So by definition of complement, \( x \notin B \).

Then, by definition of set difference, \( x \in A \setminus B \).

Since \( x \) was arbitrary, we can conclude that \( A \cap B' \subseteq A \setminus B \) by definition of subset.
Strong Induction
Consider the function $a(n)$ defined for $n \geq 1$ recursively as follows.

\[
a(1) = 1 \\
a(2) = 3 \\
a(n) = 2a(n - 1) - a(n - 2) \text{ for } n \geq 3
\]

Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geq 1$. 

Work on this problem with the people around you.
Remember our Strong Induction Template!

Let $P(n)$ be “(whatever you’re trying to prove)”. We show $P(n)$ holds for all $n \geq b_{\min}$ by induction on $n$.

**Base Case:** Show $P(b_{\min}), P(b_{\min+1}), \ldots, P(b_{\max})$ are all true.

**Inductive Hypothesis:** Suppose $P(b_{\min}) \land \ldots \land P(k)$ hold for an arbitrary $k \geq b_{\max}$.

**Inductive Step:** Show $P(k + 1)$ (i.e. get $P(b_{\min}) \land \ldots \land P(k) \rightarrow P(k + 1)$)

**Conclusion:** Therefore, $P(n)$ holds for all $n \geq b_{\min}$ by the principle of induction.
Consider the function $a(n)$ defined for $n \geq 1$ recursively as follows.

$$a(1) = 1$$

$$a(2) = 3$$

$$a(n) = 2a(n - 1) - a(n - 2) \text{ for } n \geq 3$$

Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geq 1$. 
Problem 6

Let $P(n)$ be “”. We show $P(n)$ holds...

**Base Cases:**

**Inductive Hypothesis:**

**Inductive Step:**

**Conclusion:** Therefore, $P(n)$ holds for all ... by the principle of induction.
Problem 6

Let $P(n)$ be “$a(n) = 2n - 1$”.
We show $P(n)$ holds...

Base Cases:
Inductive Hypothesis:

Inductive Step:

Conclusion: Therefore, $P(n)$ holds for all ... by the principle of induction
Problem 6

Let $P(n)$ be “$a(n) = 2n - 1$”.
We show $P(n)$ holds for all $n \geq 1$ by induction on $n$.

Base Cases:
Inductive Hypothesis:

Inductive Step:

Conclusion: Therefore, $P(n)$ holds for all $n \geq 1$ by the principle of induction
Problem 6

Let $P(n)$ be “$a(n) = 2n - 1$”.
We show $P(n)$ holds for all $n \geq 1$ by induction on $n$.

**Base Cases:** $(n = 1, n = 2)$ $a(1) = 1 = 2(1) - 1$ and $a(2) = 3 = 2(2) - 1$ by definition of $a$.

**Inductive Hypothesis:**

**Inductive Step:**

**Conclusion:** Therefore, $P(n)$ holds for all $n \geq 1$ by the principle of induction.


**Problem 6**

Let $P(n)$ be “$a(n) = 2n - 1$”.

We show $P(n)$ holds for all $n \geq 1$ by induction on $n$.

**Base Cases:** $(n = 1, n = 2)$  $a(1) = 1 = 2(1) - 1$ and $a(2) = 3 = 2(2) - 1$ by definition of $a$.

**Inductive Hypothesis:** Suppose $P(1) \land P(2) \land \ldots \land P(k)$ hold for an arbitrary $k \geq 2$.

i.e. $a(k) = 2k - 1$, $a(k - 1) = 2(k - 1) - 1$, $a(k - 2) = 2(k - 2) - 1$, etc.

**Inductive Step:**

**Conclusion:** Therefore, $P(n)$ holds for all $n \geq 1$ by the principle of induction.
Problem 6

Let $P(n)$ be “$a(n) = 2n - 1$”.

We show $P(n)$ holds for all $n \geq 1$ by induction on $n$.

**Base Cases:** $(n = 1, n = 2)$ $a(1) = 1 = 2(1) - 1$ and $a(2) = 3 = 2(2) - 1$ by definition of $a$.

**Inductive Hypothesis:** Suppose $P(1) \land P(2) \land \ldots \land P(k)$ hold for an arbitrary $k \geq 2$.

i.e. $a(k) = 2k - 1$, $a(k - 1) = 2(k - 1) - 1$, $a(k - 2) = 2(k - 2) - 1$, etc.

**Inductive Step:** Goal: Show $P(k + 1)$: $a(k + 1) = 2(k + 1) - 1$

**Conclusion:** Therefore, $P(n)$ holds for all $n \geq 1$ by the principle of induction.
Let $P(n)$ be "$a(n) = 2n - 1$".

We show $P(n)$ holds for all $n \geq 1$ by induction on $n$.

**Base Cases:** $(n = 1, n = 2)$ $a(1) = 1 = 2(1) - 1$ and $a(2) = 3 = 2(2) - 1$ by definition of $a$.

**Inductive Hypothesis:** Suppose $P(1) \land P(2) \land \ldots \land P(k)$ hold for an arbitrary $k \geq 2$.

i.e. $a(k) = 2k - 1$, $a(k - 1) = 2(k - 1) - 1$, $a(k - 2) = 2(k - 2) - 1$, etc.

**Inductive Step:** Goal: Show $P(k + 1)$: $a(k + 1) = 2(k + 1) - 1$

$a(k + 1) = \ldots$

\[ \ldots \]

\[ = 2(k + 1) - 1 \]

**Conclusion:** Therefore, $P(n)$ holds for all $n \geq 1$ by the principle of induction.
Let $P(n)$ be “$a(n) = 2n - 1$”.

We show $P(n)$ holds for all $n \geq 1$ by induction on $n$.

**Base Cases:** $(n = 1, n = 2)$ $a(1) = 1 = 2(1) - 1$ and $a(2) = 3 = 2(2) - 1$ by definition of $a$.

**Inductive Hypothesis:** Suppose $P(1) \land P(2) \land \ldots \land P(k)$ hold for an arbitrary $k \geq 2$.

i.e. $a(k) = 2k - 1$, $a(k - 1) = 2(k - 1) - 1$, $a(k - 2) = 2(k - 2) - 1$, etc.

**Inductive Step:** Goal: Show $P(k + 1)$: $a(k + 1) = 2(k + 1) - 1$

$a(k + 1) = 2a(k) - a(k - 1)$ [Definition of $a$]

\[ ...
\]

\[ = 2(k + 1) - 1 \]

**Conclusion:** Therefore, $P(n)$ holds for all $n \geq 1$ by the principle of induction.
Problem 6

Let $P(n)$ be “$a(n) = 2n - 1$”. We show $P(n)$ holds for all $n \geq 1$ by induction on $n$.

Base Cases: $(n = 1, n = 2)$ $a(1) = 1 = 2(1) - 1$ and $a(2) = 3 = 2(2) - 1$ by definition of $a$.

Inductive Hypothesis: Suppose $P(1) \land P(2) \land \ldots \land P(k)$ hold for an arbitrary $k \geq 2$.

i.e. $a(k) = 2k - 1$, $a(k - 1) = 2(k - 1) - 1$, $a(k - 2) = 2(k - 2) - 1$, etc.

Inductive Step: Goal: Show $P(k + 1)$: $a(k + 1) = 2(k + 1) - 1$

$$a(k + 1) = 2a(k) - a(k - 1)$$ [Definition of $a$]
$$= 2(2k - 1) - (2(k - 1) - 1)$$ [Inductive Hypothesis]
$$\ldots$$
$$= 2(k + 1) - 1$$

Conclusion: Therefore, $P(n)$ holds for all $n \geq 1$ by the principle of induction.
Let \( P(n) \) be “\( a(n) = 2n - 1 \)”. We show \( P(n) \) holds for all \( n \geq 1 \) by induction on \( n \).

**Base Cases:** \((n = 1, n = 2)\) \( a(1) = 1 = 2(1) - 1 \) and \( a(2) = 3 = 2(2) - 1 \) by definition of \( a \).

**Inductive Hypothesis:** Suppose \( P(1) \land P(2) \land \ldots \land P(k) \) hold for an arbitrary \( k \geq 2 \).

i.e. \( a(k) = 2k - 1, a(k - 1) = 2(k - 1) - 1, a(k - 2) = 2(k - 2) - 1, \) etc.

**Inductive Step:** Goal: Show \( P(k + 1): a(k + 1) = 2(k + 1) - 1 \)

\[
a(k + 1) = 2a(k) - a(k - 1) \quad \text{[Definition of } a]\n\]
\[
= 2(2k - 1) - (2(k - 1) - 1) \quad \text{[Inductive Hypothesis]}\n\]
\[
= 2k + 1 \quad \text{[Algebra]}\n\]
\[
= 2(k + 1) - 1 \quad \text{[Algebra]}\n\]

**Conclusion:** Therefore, \( P(n) \) holds for all \( n \geq 1 \) by the principle of induction.
Questions?

Topics:
- Translations & Predicate Logic
- English Proofs
- Number Theory
- Set Theory
- Strong Induction
- Weak Induction
That’s All Folks!

Breathe, you are going to do great!