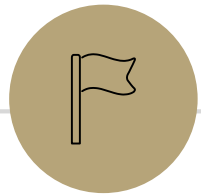


# Backwards Proofs





# Logical Ordering

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# Logical Ordering

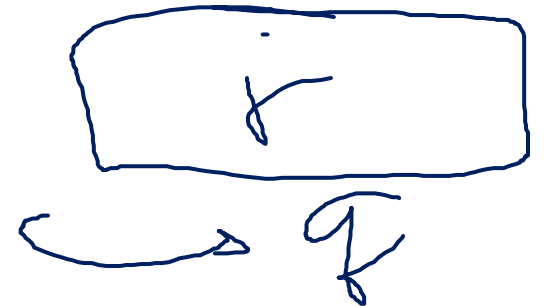
When doing a proof, we often work from both sides...

But we have to be careful!

When you read from top to bottom, every step has to follow only from what's **before** it, not after it.

Suppose our target is  $q$  and I know  $q \rightarrow p$  and  $r \rightarrow q$ .

What can I put as a "new target?"



# Logical Ordering

So why have all our prior steps been ok backward?

They've all been either:

A definition (which is always an "if and only if")

An algebra step that is an "if and only if"

Even if your steps are "if and only if" you still have to put everything in order – start from your assumptions, and only assert something once it can be shown.

# A bad proof

Claim: if  $x$  is positive then  $x + 5 = -x - 5$ .

$$\rightarrow x + 5 = -x - 5$$

$$|x + 5| = |-x - 5|$$

$$|x + 5| = |-(x + 5)|$$

$$|x + 5| = |x + 5|$$

$$\underline{0 = 0}$$

This claim is **false** – if you're trying to do algebra, you need to start with an equation you know (say  $x = x$  or  $2 = 2$  or  $0 = 0$ ) and expand to the equation you want.

$x \neq 5$

$$-(x + 5)$$

$$0 = 0$$

$$|x + 5| = |x + 5|$$
$$|x + 5| = |-(x + 5)|$$