Full outline

1. Suppose for the sake of contradiction that \( L \) is regular. Then there is some DFA \( M \) that recognizes \( L \).

2. Let \( S \) be [fill in with an infinite set of prefixes].

3. Because the DFA is finite and \( S \) is infinite, there are two (different) strings \( x, y \) in \( S \) such that \( x \) and \( y \) go to the same state when read by \( M \) [you don't get to control \( x, y \) other than having them not equal and in \( S \)].

4. Consider the string \( z \) [argue exactly one of \( xz, yz \) will be in \( L \)].

5. Since \( x, y \) both end up in the same state, and we appended the same \( z \), both \( xz \) and \( yz \) end up in the same state of \( M \). Since \( xz \in L \) and \( yz \notin L \), \( M \) does not recognize \( L \). But that's a contradiction!

6. So \( L \) must be an irregular language.

Bijection

**One-to-one (aka injection)**

A function \( f \) is one-to-one iff
\[
\forall a \forall b (f(a) = f(b) \rightarrow a = b)
\]

**Onto (aka surjection)**

A function \( f: A \rightarrow B \) is onto iff
\[
\forall b \in B \exists a \in A (b = f(a))
\]

**Bijection**

A function \( f: A \rightarrow B \) is a bijection iff
\[
f \text{ is one-to-one and onto}
\]

A bijection maps every element of the domain to exactly one element of the co-domain, and every element of the domain to exactly one element of the domain.
What do real numbers look like

0. 3 3 3 3 3 3 3 3 3...  
0. 2 7 2 7 2 8 5 4...  
0. 1 4 1 5 9 2 6 5...  
0. 2 2 2 2 2 2 2 2 2...  
0. 1 2 3 4 5 6 7 8...  
0. 9 8 7 6 5 4 3 2...  
0. 8 2 7 6 4 5 7 4...  
0. 5 9 4 2 7 5 1 7...

A string of digits!

Well not a “string” An infinitely long sequence of digits is more accurate.

Proof that [0,1) is not countable

Suppose, for the sake of contradiction, that there is a list of them:

<table>
<thead>
<tr>
<th>Number</th>
<th>Digits after decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>f(0)</td>
<td>0</td>
</tr>
<tr>
<td>f(1)</td>
<td>0</td>
</tr>
<tr>
<td>f(2)</td>
<td>0</td>
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<td>f(3)</td>
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<td>f(4)</td>
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<td>f(5)</td>
<td>0</td>
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<td>f(6)</td>
<td>0</td>
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<tr>
<td>f(7)</td>
<td>0</td>
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<td>...</td>
<td>...</td>
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</tbody>
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