Try a few of your own

Decide whether each of these relations are reflexive, symmetric, antisymmetric, and transitive.

\( \subseteq \) on \( \mathcal{P}(\mathcal{U}) \)

\( \geq \) on \( \mathbb{Z} \)

\( > \) on \( \mathbb{R} \)

\( | \) on \( \mathbb{Z}^+ \)

\( | \) on \( \mathbb{Z} \)

\( \equiv \) (mod 3) on \( \mathbb{Z} \)

Two Prototype Relations

A lot of fundamental relations follow one of two prototypes:

**Equivalence Relation**

A relation that is reflexive, symmetric, and transitive is called an “equivalence relation”

**Partial Order Relation**

A relation that is reflexive, antisymmetric, and transitive is called a “partial order”
Directed Graphs

\[ G = (V, E) \]

\( V \) is a set of vertices (an underlying set of elements)

\( E \) is a set of edges (ordered pairs of vertices; i.e. connections from one to the next).

Path \( v_0, v_1, ... , v_k \) such that \( (v_i, v_{i+1}) \in E \)

Simple Path: path with all \( v_i \) distinct

Cycle: path with \( v_0 = v_k \) (and \( k > 0 \))

Simple Cycle: simple path plus edge \( (v_k, v_0) \) with \( k > 0 \)

Relations and Graphs

Describe how each property will show up in the graph of a relation.

Reflexive

Symmetric

Antisymmetric

Transitive