Back to the arithmetic

\[ E \rightarrow E + E | E * E | (E) | x | y | z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 \]

Two parse trees for \(2 + 3 * 4\)

Takeaways

CFGs and regular expressions gave us ways of succinctly representing sets of strings
Regular expressions super useful for representing things you need to search for
CFGs represent complicated languages like “java code with valid syntax”

This week, two more tools for our toolbox (relations, graphs)
After Thanksgiving, (mathematical representations of) Tiny computers!
And how they relate to regular expressions and CFGs.
Relations

A (binary) relation from $A$ to $B$ is a subset of $A \times B$.
A (binary) relation on $A$ is a subset of $A \times A$.

Wait what?

$\leq$ is a relation on $\mathbb{Z}$.

“$3 \leq 4$” is a way of saying “$3$ relates to $4$” (for the $\leq$ relation).

$(3,4)$ is an element of the set that defines the relation.

Try a few of your own

Decide whether each of these relations are
Reflexive, symmetric, antisymmetric, and transitive.

$\subseteq$ on $\mathcal{P}(\mathcal{U})$

Symmetry: for all $a, b \in S$, $[(a,b) \in R \rightarrow (b,a) \in R]$.

$\geq$ on $\mathbb{Z}$

Antisymmetry: for all $a, b \in S$, $[(a,b) \in R \land a \neq b \rightarrow (b,a) \notin R]$.

$>$ on $\mathbb{R}$

Transitivity: for all $a, b, c \in S$, $[(a,b) \in R \land (b,c) \in R \rightarrow (a,c) \in R]$.

$|\ |$ on $\mathbb{Z}^+$

Reflexivity: for all $a \in S$, $[(a,a) \in R]$.

$|\ |$ on $\mathbb{Z}$

$\equiv (mod\ 3)$ on $\mathbb{Z}$.