Regular Expressions

Basis:

ε is a regular expression. The empty string itself matches the pattern (and nothing else does).

$\emptyset$ is a regular expression. No strings match this pattern.

a is a regular expression, for any $a \in \Sigma$ (i.e. any character). The character itself matching this pattern.

Recursive

If A, B are regular expressions then $(A \cup B)$ is a regular expression
  matched by any string that matches A or that matches B [or both]).

If A, B are regular expressions then AB is a regular expression.
  matched by any string x such that $x = yz$, y matches A and z matches B.

If A is a regular expression, then A^* is a regular expression.
  matched by any string that can be divided into 0 or more strings that match A.

More Examples

$(0^*1^*)^*$

0^*1^*

$(0 \cup 1)^*(00 \cup 11)^*(0 \cup 1)^*$

$(00 \cup 11)^*$

More Practice

You can also go the other way

Write a regular expression for "the set of all binary strings of odd length"

Write a regular expression for "the set of all binary strings with at most two ones"

Write a regular expression for "strings that don't contain 00"

Induction: Hats!

Define $P(n)$ to be "in every line of n people with gold and purple hats, with a purple hat at one end and a gold hat at the other, there is a person with a purple hat next to someone with a gold hat"

We show $P(n)$ for all integers $n \geq 2$ by induction on n.

Base Case: $n = 2$

Inductive Hypothesis:

Inductive Step:

By the principle of induction, we have $P(n)$ for all $n \geq 2$