

Let's Try Another! Stamp Collecting

I have 4 cent stamps and 5 cent stamps (as many as I want of each).
Prove that I can make exactly n cents worth of stamps for all $n \geq 12$.

Try for a few values.

Then think...how would the inductive step go?



Stamp Collection (attempt)

Define $P(n)$ I can make n cents of stamps with just 4 and 5 cent stamps.
We prove $P(n)$ is true for all $n \geq 12$ by induction on n .

Base Case:

12 cents can be made with three 4 cent stamps.

Inductive Hypothesis Suppose [maybe some other stuff and] $P(k)$, for an arbitrary $k \geq 12$.

Inductive Step:

We want to make $k + 1$ cents of stamps. By IH we can make $k - 3$ cents exactly with stamps. Adding another 4 cent stamp gives exactly $k + 1$ cents.

Stamp Collection, Done Wrong

Define $P(n)$ I can make n cents of stamps with just 4 and 5 cent stamps.

We prove $P(n)$ is true for all $n \geq 12$ by induction on n .

Base Case:

12 cents can be made with three 4 cent stamps.

Inductive Hypothesis Suppose $P(k)$, $k \geq 12$.

Inductive Step:

We want to make $k + 1$ cents of stamps. By IH we can make k cents exactly with stamps. Replace one of the 4 cent stamps with a 5 cent stamp.

$P(n)$ holds for all n by the principle of induction.

Fibonacci Inequality

Show that $f(n) \leq 2^n$ for all $n \geq 0$ by induction.

$$f(0) = 1; \quad f(1) = 1$$

$$f(n) = f(n-1) + f(n-2) \text{ for all } n \in \mathbb{N}, n \geq 2.$$