Divides

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For integers x, y we say $x|y$ ("x divides y") iff there is an integer z such that $xz = y$.

Which of these are true?

$2|4$ $\quad$ $4|2$ $\quad$ $2|-2$

$5|0$ $\quad$ $0|5$ $\quad$ $1|5$

A useful theorem

The Division Theorem

For every $a \in \mathbb{Z}$, $d \in \mathbb{Z}$ with $d > 0$
There exist *unique* integers q, r with $0 \leq r < d$
Such that $a = dq + r$

Remember when non integers were still secret, you did division like this?

$$\begin{array}{r} 4 \text{ R}5 \\ 7\overline{)33} \\ \underline{28} \\ 5 \end{array}$$

q is the "quotient"
r is the "remainder"

Claim: for all $a, b, c, n \in \mathbb{Z}, n > 0$: $a \equiv b \pmod{n} \rightarrow a + c \equiv b + c \pmod{n}$

Before we start, we must know:

1. What every word in the statement means.
2. What the statement as a whole means.
3. Where to start.
4. What your target is.

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Equivalence in modular arithmetic

Let $a \in \mathbb{Z}, b \in \mathbb{Z}, n \in \mathbb{Z}$ and $n > 0$.
We say $a \equiv b \ (mod\ n)$ if and only if $n|(b - a)$

Another Proof

For all integers, a, b, c: Show that if $a \nmid (bc)$ then $a \nmid b$ or $a \nmid c$.

Proof:

Let a, b, c be arbitrary integers, and suppose $a \nmid (bc)$.

Then there is not an integer z such that $az = bc$

…

So $a \nmid b$ or $a \nmid c$