Inference Rules

Eliminate ∧

$$\frac{A \land B}{\therefore\ A, B}$$

Eliminate ∨

$$\frac{A \lor B, \neg A}{\therefore\ B}$$

Intro ∧

$$\frac{A, B}{\therefore\ A \land B}$$

Intro ∨

$$\frac{A}{\therefore A \lor B, B \lor A}$$

Direct Proof rule

$$\frac{A \Rightarrow B}{A \rightarrow B}$$

Modus Ponens

$$\frac{P \rightarrow Q, P}{\therefore\ Q}$$

You can still use all the propositional logic equivalences too!

Try it!

Given: $p \lor q$, $(r \land s) \rightarrow \neg q$, r.
Show: $s \rightarrow p$

Eliminate ∧

$$\frac{A \land B}{\therefore\ A, B}$$

Eliminate ∨

$$\frac{A \lor B, \neg A}{\therefore\ B}$$

Intro ∧

$$\frac{A; B}{\therefore\ A \land B}$$

Intro ∨

$$\frac{A}{\therefore A \lor B, B \lor A}$$

Direct Proof rule

$$\frac{A \Rightarrow B}{A \rightarrow B}$$

Modus Ponens

$$\frac{P \rightarrow Q; P}{\therefore\ Q}$$

You can still use all the propositional logic equivalences too!

Proof Using Quantifiers

Suppose we know $\exists x P(x)$ and $\forall y[\, P(y) \rightarrow Q(y)]$. Conclude $\exists x Q(x)$.

Intro ∃

$$\frac{P(c) \text{ for some } c}{\therefore \quad \exists x\, P(x)}$$

Eliminate ∃

$$\frac{\exists x P(x)}{\therefore \quad P(c) \text{ for a fresh } c}$$

Eliminate ∀

$$\frac{\forall x\, P(x)}{\therefore P(a) \text{ for any } a}$$

Intro ∀

$$\frac{P(a);\, a \text{ is arbitrary}}{\therefore \quad \forall x\, P(x)}$$

Arbitrary

In section, you said: $[\exists y \forall x\, P(x, y)] \rightarrow [\forall x \exists y\, P(x, y)]$. Let's prove it!!

1.1 $\exists y \forall x\, P(x, y)$	Assumption
1.2 $\forall x\, P(x, c)$	Elim ∃ (1.1)
1.3 Let a be arbitrary.	--
1.4 $P(a, c)$	Elim ∀ (1.2)
1.5 $\exists y\, P(a, y)$	Intro ∃ (1.4)
1.6 $\forall x \exists y\, P(x, y)$	Intro ∀ (1.5)
2. $[\exists y \forall x\, P(x, y)] \rightarrow [\forall x \exists y\, P(x, y)]$	Direct Proof Rule