Direct Proof Steps

These are the usual steps. We'll see different outlines in the future!!

- Introduction
  - Declare an arbitrary variable for each $\forall$ quantifier
  - Assume the left side of the implication
- Core of the proof
  - Unroll the predicate definitions
  - Manipulate towards the goal (using creativity, algebra, etc.)
  - Reroll definitions into the right side of the implication
- Conclude that you have proved the claim

Another Direct Proof

Prove: “The product of two odd integers is odd.”

What’s the claim in logic?

How would we prove this claim?
Yet Another Direct Proof

Prove: “The product of two square integers is square.”

$$\forall n \forall m \left( \left( \text{Square}(n) \land \text{Square}(m) \right) \rightarrow \text{Square}(nm) \right)$$

Definitions

Square(x) := \exists k \ (x = k^2)

Try it yourselves

Suppose you know $p \rightarrow q$, $\neg s \rightarrow \neg q$, and $p$.
Give an argument to conclude $s$.