Nested Quantifiers

Let our domain of discourse be $\{A, B, C, D, E\}$

And our proposition $P(x, y)$ be given by the table.

What should we look for in the table?

$\exists x \forall y P(x, y)$

$\forall x \exists y P(x, y)$

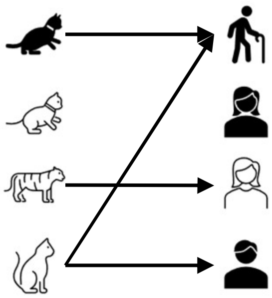
y

x

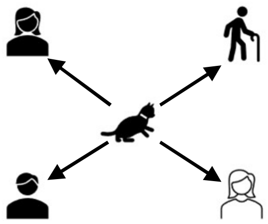
$P(x,y)$	A	B	C	D	E
A	T	T	T	T	T
B	T	F	F	T	F
C	F	T	F	F	F
D	F	F	F	F	T
E	F	F	F	T	F

Try it yourselves

Every cat loves some human.



There is a cat that loves every human.



Let your domain of discourse be mammals.
Use the predicates `Cat`(x), `Dog`(x), and `Loves`(x, y) to mean x loves y.

To “limit” variables to a portion of your domain of discourse under a universal quantifier add a hypothesis to an implication.

To “limit” variables to a portion of your domain of discourse under an existential quantifier AND the limitation together with the rest of the statement.

To negate an expression with a quantifier
1. Switch the quantifier (∀ becomes ∃, ∃ becomes ∀)
2. Negate the expression inside

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1. The statement is true for every x, we just want to put a name on it.

$\forall x\ (\mathrm{p}(\mathrm{x}) \land q(x))$ means “for every x in our domain, $p(x)$ and $q(x)$ both evaluate to true.”

Universal Quantifier

“$\forall x$“

“for each x”, “for every x”, “for all x” are common translations
Remember: upside-down-A for All.

2. There’s some x out there that works, (but I might not know which it is, so I’m using a variable).

$\exists x(p(x) \land q(x))$ means “there is an x in our domain, $p(x)$ and $q(x)$ are both true.

Existential Quantifier

“$\exists x$“

“there is an x”, “there exists an x”, “for some x” are common translations
Remember: backwards-E for Exists.