

## Our First Proof

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv$$

None of the rules look like this

Practice of Proof-Writing:

**Big Picture**...WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines...maybe the " $\neg a$ "  $\equiv (\neg a \vee b)$  came from there? Maybe that **simplifies** down to  $\neg a$

## Simplify $\top \wedge (\neg a \vee b)$ to $(\neg a \vee b)$

These identities hold for all propositions  $p, q, r$

- **Identity**
  - $p \wedge \top \equiv p$
  - $p \vee \text{F} \equiv p$
- **Domination**
  - $p \vee \top \equiv \top$
  - $p \wedge \text{F} \equiv \text{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv \top$
  - $p \wedge \neg p \equiv \text{F}$

## Vocabulary!

**A proposition is a....**

*Tautology* if it is always true.

*Contradiction* if it is always false.

*Contingency* if it can be both true and false.

$$p \vee \neg p$$

Tautology

If  $p$  is true,  $p \vee \neg p$  is true; if  $p$  is false,  $p \vee \neg p$  is true.

$$p \oplus p$$

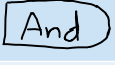

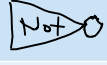
Contradiction

If  $p$  is true,  $p \oplus p$  is false; if  $p$  is false,  $p \oplus p$  is false.

$$(p \rightarrow q) \wedge p$$

Contingency If  $p$  is true and  $q$  is true,  $(p \rightarrow q) \wedge p$  is true;  
If  $p$  is true and  $q$  is false,  $(p \rightarrow q) \wedge p$  is false.

## Meet Boolean Algebra

| Name                | Variables | "True/False" | "And"   | "Or"   | "Not"   | Implication       |
|---------------------|-----------|--------------|---|--|---|-------------------|
| Java Code           | boolean b | true, false  | &&  |  | !   | No special symbol |
| Propositional Logic | "p, q, r" | T, F         | $\wedge$  | $\vee$   | $\neg$  | $\rightarrow$     |
| Circuits            | Wires     | 1, 0         |  |  |  | No special symbol |
| Boolean Algebra     | $a, b, c$ | 1, 0         | $\cdot$<br>("multiplication")   | $+$<br>("addition")  | '<br>(apostrophe after variable)  | No special symbol |

Propositional logic

$$(p \wedge q \wedge r) \vee s \vee \neg t$$

Boolean Algebra

$$pqr + s + t'$$