A More Complicated Statement

“Robbie knows the Pythagorean Theorem if he is a mathematician and took geometry, and he is a mathematician or did not take geometry.”

Is this a proposition?

We’d like to understand what this proposition means.

In particular, is it true?

Law of Implication

Implications are hard. AND/OR/NOT make more intuitive sense to me... can we rewrite implications using just ANDs ORs and NOTs?

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One approach: think “when is this implication false?” then negate it (you might want one of DeMorgan’s Laws!)
Properties of Logical Connectives

These identities hold for all propositions $p, q, r$

- **Identity**
  - $p \land T \equiv p$
  - $p \lor F \equiv p$
- **Domination**
  - $p \lor T \equiv T$
  - $p \land F \equiv F$
- **Idempotent**
  - $p \lor p \equiv p$
  - $p \land p \equiv p$
- **Commutative**
  - $p \lor q \equiv q \lor p$
  - $p \land q \equiv q \land p$
- **Associative**
  - $(p \lor q) \lor r \equiv p \lor (q \lor r)$
  - $(p \land q) \land r \equiv p \land (q \land r)$
- **Distributive**
  - $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
  - $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
- **Absorption**
  - $p \lor (p \land q) \equiv p$
  - $p \land (p \lor q) \equiv p$
- **Negation**
  - $p \lor \neg p \equiv T$
  - $p \land \neg p \equiv F$

Our First Proof

$$(a \land b) \lor (\neg a \land b) \lor (\neg a \land \neg b) \equiv$$

None of the rules look like this

**Practice of Proof-Writing:**

**Big Picture**...WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines...maybe the "$\neg a$" came from there? Maybe that **simplifies** down to $\neg a$