CSE 311 : Practice Final

This exam is a (slight) modification of a real final given in a prior quarter of CSE311.

The original exam was given in a 110 minute slot.

We strongly recommend you take this exam as though it were closed book – even though your exam will be open book.

Instructions

- Students had 110 minutes to complete the exam.
- The exam was closed resource (except for the logical equivalences, boolean algebra, and inference rules reference sheets). Your exam will allow a notes sheet.
- The problems are of varying difficulty.
- If you get stuck on a problem, move on and come back to it later.

1. Aquatic Translation [10 points]

Translate the following sentences into logical notation if the English statement is given or to an English statement if the logical statement is given, taking into account the domain restriction. Let the domain of discourse be sea creatures and seas. Use predicates Fish and Sea to do the domain restriction. You can use $\texttt{SwimIn}(x, y)$ which is true if and only if x can swim in y. The predicate $\texttt{NarwhalsLive}(x)$ is true if and only if narwhals live in x, and $\texttt{ContainsIcebergs}(x)$ is true if and only if x contains icebergs.

(a) Every fish can swim in some sea.

(b) There is a fish that cannot swim in every sea.

(c) Some fish can only swim in one sea.

(d) $\forall x[(\texttt{Sea}(x) \land \texttt{NarwhalsLive}(x)) \rightarrow \texttt{ContainsIcebergs}(x)]$

(e) $\exists x(\texttt{Sea}(x) \land \texttt{NarwhalsLive}(x) \land \texttt{ContainsIcebergs}(x) \land \forall y[(\texttt{Sea}(y) \land \texttt{NarwhalsLive}(y)) \rightarrow (x = y)])$

2. Regularly Irregular [15 points]

Let $\Sigma = \{0, 1\}$. Prove that the language $L = \{x \in \Sigma^* : \#_0(x) < \#_1(x)\}$ is irregular.

3. Recurrences, Recurrences [15 points]

Define

$$T(n) = \begin{cases} n & \text{if } n = 0, 1 \\ 4T\left(\lfloor \frac{n}{2} \rfloor\right) + n & \text{otherwise} \end{cases}$$

Prove that $T(n) \leq n^3$ for $n \geq 3$.

4. Structural CFGs [15 points]

Consider the following CFG: **S** $\to \varepsilon$ | **SS** | **S**1 | **S**01. Another way of writing the recursive definition of this set, Q, is as follows:

- $\varepsilon \in Q$
- If $S \in Q$, then $S1 \in Q$ and $S01 \in Q$
- If $S, T \in Q$, then $ST \in Q$.

Prove, by structural induction that if $w \in Q$, then w has at least as many 1's as 0's.

5. Tralse! [15 points]

For each of the following answer True or False and give a short explanation of your answer.

(a) Any subset of a regular language is also regular.

(b) The set of programs that loop forever on at least one input is decidable.

(c) If $\mathbb{R} \subseteq A$ for some set A, then A is uncountable.

(d) If the domain of discourse is people, the logical statement

$$\exists x\ (P(x) \rightarrow \forall y\ (x \neq y \rightarrow \neg P(y))$$

can be correctly translated as "There exists a unique person who has property P".

(e) $\exists x\ (\forall y\ P(x, y)) \rightarrow \forall y\ (\exists x\ P(x, y))$ is true regardless of what predicate P is.

6. Relationships and Sets! [15 points]

(a) Let $S = \{(X, Y) : X, Y \in \mathcal{P}(\mathbb{N}) \wedge X \subseteq Y\}$.

Recall that R is antisymmetric iff $((a, b) \in R \wedge a \neq b) \rightarrow (b, a) \notin R$.

Prove that S is antisymmetric.

(b) Let $A = \{x : x \equiv k (\mod 4)\}$, $B = \{x : x = 4r + k \text{ for some integer } r\}$. Prove that $A = B$ for all integers k.

7. All The Machines! [15 points]

Let $\Sigma = \{0, 1, 2\}$.
Consider $L = \{w \in \Sigma^* :$ Every 1 in the string has at least one 0 before and after it$\}$.
Note that the 0s need not be directly adjacent to the 1s.

(a) Give a regular expression that represents A.

(b) Give a DFA that recognizes A.

(c) Give a CFG that generates A.

(d) Let $\Sigma = \{0, 1, 2\}$. Construct a DFA that recognizes exactly strings with a 0 in all positions i where $i\%3 = 0$.