CSE 311 : 22Wi Midterm Exam

Directions

This is a pdf version of the 22Wi 311 midterm exam. The exam was originally delivered on gradescope.

The exam was intended to be solvable in close to 30 minutes, but the time limit was 2 hours to give plenty of buffer.

1. Distributed Systems + NFT = Profit??

For the following statements, translate them into predicate logic, keeping in mind the respective domains of discourse and predicates.

(a) Translate the following statement into logic:

"Robbie owns exactly two distinct NFTs."

Let your domain of discourse be all NFTs. You should use the following predicates:

- $\texttt{RobbieOwns}(x)$ is true iff x is an NFT owned by Robbie
- $\texttt{Equals}(x, y)$ is true iff the two objects are equal (as in the exact same object)

(b) Translate the following statement into logic:

"Modern day systems which are partition tolerant cannot be both consistent and available."

Let your domain of discourse be all systems. You should use the following predicates:

- $\texttt{ModernDaySystem}(x)$ is true iff x is a modern day system
- $\texttt{PartitionTolerant}(x)$ is true iff x is partition tolerant
- $\texttt{Consistent}(x)$ is true iff x is consistent
- $\texttt{Available}(x)$ is true iff x is available

(c) Let your domain of discourse be all integers (i.e., $\mathbb{Z}$). We define the following predicates:

- $\texttt{Perfect}(x)$ is true iff x is a perfect number
- $\texttt{Even}(x)$ is true iff x is even
- $\texttt{Greater}(a, b)$ is true iff $a > b$

Consider the following statement:

$$\forall x(\texttt{Perfect}(x) \rightarrow [\texttt{Even}(x) \vee \texttt{Greater}(x, 1000)])$$

Write (in predicate logic notation) an equivalent statement to the one above by taking the contrapositive of the implication. Your notation should be simplified, by having negations apply only to individual predicates.

Then, translate your statement into English. You do not have to use domain restriction in your translation.

2. Non-Inductible Tokens [20 points]

In his eternal pursuit for the ideal NFT, Robbie has decided to focus on creating some NFTs with various levels of scarcity. Every day, Robbie creates one new NFT and adds it to his collection.

Define the scarcity of day n's NFT as $\mathcal{S}_n = 0+1+\cdots+n$ where $n \in \mathbb{N}$. We proved in section that $\mathcal{S}_n = \frac{n(n+1)}{2}$.

Now, define the combined scarcity of the NFTs in Robbie's wallet on day n as $\mathcal{T}_n = \mathcal{S}_0 + \mathcal{S}_1 + \cdots + \mathcal{S}_n$, where $n \in \mathbb{N}$.

Now, it's your turn! Prove by way of induction for all $n \in \mathbb{N}$:

$$\mathcal{T}_0 + \mathcal{T}_1 + \cdots + \mathcal{T}_n = \frac{n(n+1)(n+2)(n+3)}{24}.$$

You may (and should) use the following as fact: $\mathcal{T}_n = \frac{n(n+1)(n+2)}{6}$.

IMPORTANT: There is NO NEED to expand out/"FOIL" out the expression when doing your inductive step! We only need to factor out expressions.

An example of expanding/FOILing is going from $(x+y)(5y+z)$ to $5xy + xz + 5y^2 + yz$.
An example of factoring is: $5x + xy = x(5+y)$.

3. NFT's $\neq$ Boring

(a) Translate the following statement into logic:

> There is a "neutral number", such that any real number a when added to the neutral number yields a.

Let your domain of discourse be all real numbers (i.e., $\mathbb{R}$). You may use the mathematical operator $+$ and quantifiers ($\exists$, $\forall$). You can *only* use the following predicate:

- `Equals`(a, b) is true iff $a = b$

You may not define a predicate "neutral" you must incorporate that meaning directly with just `Equals`(a, b).

(b) We call an integer b "**boring** for $\pmod{n}$ arithmetic" iff for all a: $a + b \equiv a \pmod{n}$, where $a, b \in \mathbb{Z}$.

Show that for integers b and b' where both b and b' are boring for $\pmod{n}$ arithmetic, that $b \equiv b' \pmod{n}$. For this problem you may **not** use any of the identities on the number theory reference sheet.

You must complete this proof using only the definitions of mod and divides and algebra.

For example, you cannot subtract a from both sides of a **modular** equation (as that would be applying the "adding congruences" theorem), but you can subtract a from both sides of a "regular = equation," as that's "just algebra."

A reminder of definitions:

- $a \equiv b \pmod{m}$ iff $m \mid (b - a)$ (def of mod)
- $a \mid b$ iff $\exists (k \in \mathbb{Z})\ b = ka$ (def of divides)