

Section 09: CFGs, Relations, DFAs, NFAs, and Minimization

1. CFGs

Write a context-free grammar to match each of these languages.

- (a) All binary strings that start with 11.

- (b) All binary strings that contain at most one 1.

- (c) All strings over 0, 1, 2 with the same number of 1s and 0s and exactly one 2.
Hint: Try modifying the grammar from Section 8 2c for binary strings with the same number of 1s and 0s (You may need to introduce new variables in the process).

2. Relations

- (a) Draw the transitive-reflexive closure of $\{(1, 2), (2, 3), (3, 4)\}$.

- (b) Suppose that R is reflexive. Prove that $R \subseteq R^2$.

- (c) Consider the relation $R = \{(x, y) : x = y + 1\}$ on \mathbb{N} . Is R reflexive? Transitive? Symmetric? Anti-symmetric?

- (d) Consider the relation $S = \{(x, y) : x^2 = y^2\}$ on \mathbb{R} . Prove that S is reflexive, transitive, and symmetric.

3. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1, 2, 3\}$.

- (a) All binary strings.

- (b) All strings whose digits sum to an even number.

- (c) All strings whose digits sum to an odd number.

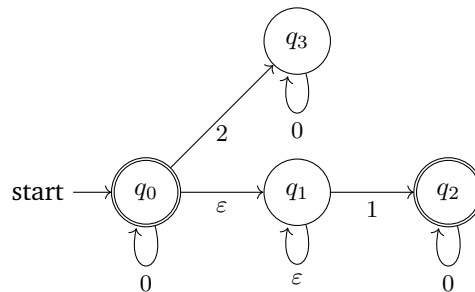
4. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

- All strings which do not contain the substring 101.
- All strings containing at least two 0's and at most one 1.
- All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

5. NFAs

- What language does the following NFA accept?



- Create an NFA for the language “all binary strings that have a 1 as one of the last three digits”.

6. DFAs & Minimization

Note: We will not test you on minimization, although you may optionally read the extra slides and do this problem for fun

- Convert the NFA from 1a to a DFA, then minimize it.
- Minimize the following DFA:

