Section 04: Propositions and Proofs

1. Formal Spoofs

For each of the following proofs, determine why the proof is incorrect. Then, consider whether the conclusion of the proof is true or not. If it is true, state how the proof could be fixed. If it is false, give a counterexample.

(a) Show that $\exists z\ \forall x\ P(x, z)$ follows from $\forall x\ \exists y\ P(x, y)$.

1.	$\forall x\ \exists y\ P(x, y)$	[Given]
2.	$\forall x\ P(x, c)$	[$\exists$ Elim: 1, c special]
3.	$\exists z\ \forall x P(x, z)$	[$\exists$ Intro: 2]

(b) Show that $\exists z\ (P(z) \wedge Q(z))$ follows from $\forall x\ P(x)$ and $\exists y\ Q(y)$.

1.	$\forall x\ P(x)$	[Given]
2.	$\exists y\ Q(y)$	[Given]
3.	Let z be arbitrary	
4.	$P(z)$	[$\forall$ Elim: 1]
5.	$Q(z)$	[$\exists$ Elim: 2, let z be the object that satisfies $Q(z)$]
6.	$P(z) \wedge Q(z)$	[$\wedge$ Intro: 4, 5]
7.	$\exists z\ P(z) \wedge Q(z)$	[$\exists$ Intro: 6]

2. How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say ∞.

(a) $A = \{1, 2, 3, 2\}$

(b) $B = \{\{\},\ \{\{\}\},\ \{\{\}, \{\}\},\ \{\{\}, \{\}, \{\}\},\ \ldots\}$

(c) $C = A \times (B \cup \{7\})$

(d) $D = \varnothing$

(e) $E = \{\varnothing\}$

(f) $F = \mathcal{P}(\{\varnothing\})$

3. Set = Set

Prove the following set identities.

(a) Let the universal set be $\mathcal{U}$. Prove $A \cap \overline{B} \subseteq A \backslash B$ for any sets A, B.

(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

4. Set Equality

(a) Let $\mathcal{U}$ be the universal set. Show that $\overline{\overline{X}} = X$.

5. Trickier Set Theory

Note, this problem requires a little more thinking. The solution will cover both the answer as well as the intuition used to arrive at it.

Show that for any set X and any set A such that $A \in \mathcal{P}(X)$, there exists a set $B \in \mathcal{P}(X)$ such that $A \cap B = \emptyset$ and $A \cup B = X$.