Bijection

One-to-one (aka injection)

A function f is one-to-one iff
$\forall a \forall b(f(a) = f(b) \rightarrow a = b)$

Onto (aka surjection)

A function $f: A \rightarrow B$ is onto iff
$\forall b \in B \exists a \in A(b = f(a))$

Bijection

A function $f: A \rightarrow B$ is a bijection iff
f is one-to-one and onto

A bijection maps every element of the domain to exactly one element of the co-domain, and every element of the domain to exactly one element of the domain.

Some infinite sets

Two sets A, B have the same size (same cardinality) if and only if there is a bijection $f: A \rightarrow B$

Let's compare the sizes of: $\mathbb{N}$, $\mathbb{Z}$, $\{x : x$ is an even integer$\}$

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Proof that [0,1) is not countable

Suppose, for the sake of contradiction, that there is a list of them:

Number	Digits after decimal	0	1	2	3	4	5	6	7	…
$f(0)$	0.	3	3	3	3	3				
$f(1)$	0.	2	7	2	7	2				
$f(2)$	0.	1	4	1	5	9				
$f(3)$	0.	2	2	2	2	2				
$f(4)$	0.	1	2	3	4	5	6	7	8	…
$f(5)$	0.	9	8	7	6	5	4	3	2	…
$f(6)$	0.	8	2	7	6	4	5	7	4	…
$f(7)$	0.	5	9	4	2	7	5	1	7	…
…	…	…	…	…	…	…	…	…	…	…

Goal: find a real number between 0 and 1 that isn't on our table.
(contradiction to bijection)

Proof that ~~[0,1)~~ set of binary-valued functions is not countable

Suppose, for the sake of contradiction, that there is a list of them:

f bijection from $\mathbb{N}$ to function		Output on 0	Output on 1	Output on 2	Output on 3	Output on 4	Output on 5	Output on 6	Output on 7	…
$f(0)$		1	0	1	1					
$f(1)$		0	1	1	0					
$f(2)$		1	1	1	0					
$f(3)$		0	0	0	0					
$f(4)$		1	0	1	1	1	0	1	1	…
$f(5)$		0	0	0	1	0	1	1	1	…
$f(6)$		1	1	0	1	0	1	1	0	…
$f(7)$		0	2	0	1	1	0	1	0	…
…		…	…	…	…	…	…	…	…	…

Goal: find a function $g_{diag}: \mathbb{N} \to \{0,1\}$ that isn't on our table.
(contradiction to bijection)