A Proof Outline

Claim: $\{0^k1^k : k \geq 0\}$ is an irregular language.

…

Let $S =$ [TODO]. *S is an infinite set of strings.*

Because the DFA is finite, there are two (different) strings x, y in S such that x and y go to the same state. *We don't get to choose x, y*

Consider the string $z =$[TODO] *We do get to choose z* depending on x, y

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that $xz \in \{0^k1^k : k \geq 0\}$ but $yz \notin \{0^k1^k : k \geq 0\}$. Since q is can be only one of an accept or reject state, M does not actually recognize $\{0^k1^k : k \geq 0\}$. That's a contradiction!

Therefore, $\{0^k1^k : k \geq 0\}$ is an irregular language.

Let's Try another

The set of strings with balanced parentheses is not regular.

What do you want S to be? What would you have to count?

The number of unclosed parentheses.

Let $S =$…

Full outline

1. Suppose for the sake of contradiction that L is regular. Then there is some DFA M that recognizes L.

2. Let S be [fill in with an infinite set of prefixes].

3. Because the DFA is finite and S is infinite, there are two (different) strings x, y in S such that x and y go to the same state when read by M *[you don't get to control x, y other than having them not equal and in S]*

4. Consider the string z [argue exactly one of xz, yz will be in L]

5. Since x, y both end up in the same state, and we appended the same z, both xz and yz end up in the same state of M. Since $xz \in L$and $yz \notin L$, M does not recognize L. But that's a contradiction!

6. So L must be an irregular language.

One more, just the key steps

What about $\{a^k b^k c^k : k \geq 0\}$?