

Properties of relations

What do we do with relations? Usually we prove properties about them.

Symmetry

A binary relation R on a set S is "symmetric" iff
for all $a, b \in S$, $[(a, b) \in R \rightarrow (b, a) \in R]$

= on Σ^* is symmetric, for all $a, b \in \Sigma^*$ if $a = b$ then $b = a$.
 \subseteq is not symmetric on $\mathcal{P}(\mathcal{U})$ – $\{1,2,3\} \subseteq \{1,2,3,4\}$ but $\{1,2,3,4\} \not\subseteq \{1,2,3\}$

Transitivity

A binary relation R on a set S is "transitive" iff
for all $a, b, c \in S$, $[(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R]$

= on Σ^* is transitive, for all $a, b, c \in \Sigma^*$ if $a = b$ and $b = c$ then $a = c$.
 \subseteq is transitive on $\mathcal{P}(\mathcal{U})$ – for any sets A, B, C if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
 \in is not a transitive relation – $1 \in \{1,2,3\}$, $\{1,2,3\} \in \mathcal{P}(\{1,2,3\})$ but $1 \notin \mathcal{P}(\{1,2,3\})$.

More Properties of relations

What do we do with relations? Usually we prove properties about them.

Antisymmetry

A binary relation R on a set S is "antisymmetric" iff
for all $a, b \in S$, $[(a, b) \in R \wedge a \neq b \rightarrow (b, a) \notin R]$

\leq is antisymmetric on \mathbb{Z}

Reflexivity

A binary relation R on a set S is "reflexive" iff
for all $a \in S$, $[(a, a) \in R]$

\leq is reflexive on \mathbb{Z}

\leq

Try a few of your own

[Pollev.com/uwcse311](https://pollev.com/uwcse311)

Decide whether each of these relations are Reflexive, symmetric, antisymmetric, and transitive.

\subseteq on $\mathcal{P}(U)$

Symmetry: for all $a, b \in S$, $[(a, b) \in R \rightarrow (b, a) \in R]$

\geq on \mathbb{Z}

Antisymmetry: for all $a, b \in S$, $[(a, b) \in R \wedge a \neq b \rightarrow (b, a) \notin R]$

$>$ on \mathbb{R}

Transitivity: for all $a, b, c \in S$, $[(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R]$

$|$ on \mathbb{Z}^+

Reflexivity: for all $a \in S$, $[(a, a) \in R]$

$|$ on \mathbb{Z}

$\equiv (\text{mod } 3)$ on \mathbb{Z}

Relations and Graphs

Describe how each property will show up in the graph of a relation.

Reflexive

Symmetric

Antisymmetric

Transitive