Context Free Grammars

We think of context free grammars as **generating** strings.

1. Start from the start symbol S.
2. Choose a nonterminal in the string, and a production rule $A \to w_1 | w_2 | \ldots | w_k$ replace that copy of the nonterminal with w_i.
3. If no nonterminals remain, you're done! Otherwise, goto step 2.

A string is in the language of the CFG iff it can be generated starting from S.

Notation: $xAy \Rightarrow xwy$ is rewriting A with w.

Arithmetic

$E \to E + E | E * E | (E) | x | y | z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

Generate $(2 * x) + y$

Generate $2 + 3 * 4$ in two different ways

pollev.com/uwcse311

Relations

Relations

A (binary) relation from A to $\boldsymbol{B}$ is a subset of $\boldsymbol{A} \times \boldsymbol{B}$
A (binary) relation on $\boldsymbol{A}$ is a subset of $\boldsymbol{A} \times \boldsymbol{A}$

Wait what?

$\leq$ is a relation on $\mathbb{Z}$.

"$3 \leq 4$" is a way of saying "3 relates to 4" (for the $\leq$ relation)

$(3,4)$ is an element of the set that defines the relation.

Properties of relations

What do we do with relations? Usually we prove properties about them.

Symmetry
A binary relation R on a set $\boldsymbol{S}$ is "symmetric" iff
for all $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{S}$, $[(\boldsymbol{a}, \boldsymbol{b}) \in \boldsymbol{R} \rightarrow (\mathbf{b}, \mathbf{a}) \in \boldsymbol{R}]$

$=$ on Σ^* is symmetric, for all $a, b \in \Sigma^*$ if $a = b$ then $b = a$.
$\subseteq$ is not symmetric on $\mathcal{P}(\mathcal{U})$ – $\{1,2,3\} \subseteq \{1,2,3,4\}$ but $\{1,2,3,4\} \nsubseteq \{1,2,3\}$

Transitivity
A binary relation R on a set $\boldsymbol{S}$ is "transitive" iff
for all $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \boldsymbol{S}$, $[(\boldsymbol{a}, \boldsymbol{b}) \in \boldsymbol{R} \wedge (\boldsymbol{b}, \boldsymbol{c}) \in \boldsymbol{R} \rightarrow (\mathbf{a}, \mathbf{c}) \in \boldsymbol{R}]$

$=$ on Σ^* is transitive, for all $a, b, c \in \Sigma^*$ if $a = b$ and $b = \mathrm{c}$ then $a = c$.
$\subseteq$ is transitive on $\mathcal{P}(\mathcal{U})$ – for any sets A, B, C if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
$\in$ is not a transitive relation – $1 \in \{1,2,3\}$, $\{1,2,3\} \in \mathcal{P}(\{1,2,3\})$ but $1 \notin \mathcal{P}(\{1,2,3\})$.