CSE 311 : Practice Final

This exam is a (slight) modification of a real final given in a prior quarter of CSE311.

The original exam was given in a 110 minute slot.

We strongly recommend you take this exam as though it were closed book – even though your exam will be open book.

Instructions

- Students had 110 minutes to complete the exam.
- The exam was closed resource (except for the logical equivalences, boolean algebra, and inference rules reference sheets). You exam will be open resource.
- The problems are of varying difficulty.
- If you get stuck on a problem, move on and come back to it later.

1. Regularly Irregular [15 points]

Let $\Sigma = \{0, 1\}$. Prove that the language $L = \{x \in \Sigma^* : \#_0(x) < \#_1(x)\}$ is irregular.

2. Recurrences, Recurrences [15 points]

Define

$$T(n) = \begin{cases} n & \text{if } n = 0, 1 \\ 4T\left(\lfloor \frac{n}{2} \rfloor\right) + n & \text{otherwise} \end{cases}$$

Prove that $T(n) \leq n^3$ for $n \geq 3$.

3. All The Machines! [15 points]

Let $\Sigma = \{0, 1, 2\}$.
Consider $L = \{w \in \Sigma^* :$ Every 1 in the string has at least one 0 before and after it$\}$.

(a) Give a regular expression that represents A.

(b) Give a DFA that recognizes A.

(c) Give a CFG that generates A.

4. Structural CFGs [15 points]

Consider the following CFG: **S** $\to \varepsilon$ | **SS** | **S**1 | **S**01. Another way of writing the recursive definition of this set, Q, is as follows:

- $\varepsilon \in Q$
- If $S \in Q$, then $S1 \in Q$ and $S01 \in Q$
- If $S, T \in Q$, then $ST \in Q$.

Prove, by structural induction that if $w \in Q$, then w has at least as many 1's as 0's.

5. Tralse! [15 points]

For each of the following answer True or False and give a short explanation of your answer.

(a) Any subset of a regular language is also regular.

(b) The set of programs that loop forever on at least one input is decidable.

(c) If $\mathbb{R} \subseteq A$ for some set A, then A is uncountable.

(d) If the domain of discourse is people, the logical statement

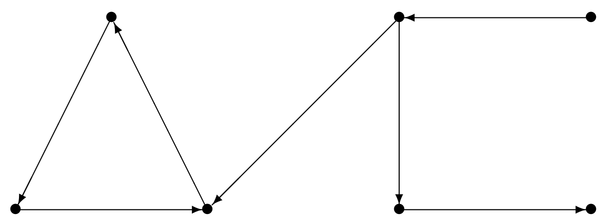
$$\exists x\ (P(x) \rightarrow \forall y\ (x \neq y \rightarrow \neg P(y))$$

can be correctly translated as "There exists a unique person who has property P".

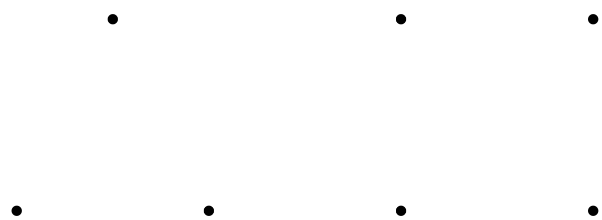
(e) $\exists x\ (\forall y\ P(x, y)) \rightarrow \forall y\ (\exists x\ P(x, y))$ is true regardless of what predicate P is.

6. Relationships! [15 points]

The following is the graph of a binary relation R.



(a) Draw the transitive-reflexive closure of R. [5 points]



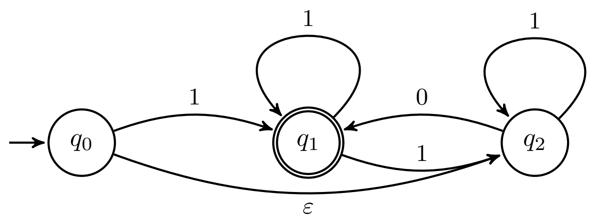
(b) Let $S = \{(X, Y) : X, Y \in \mathcal{P}(\mathbb{N}) \wedge X \subseteq Y\}$.

Recall that R is antisymmetric iff $((a, b) \in R \wedge a \neq b) \rightarrow (b, a) \notin R$.

Prove that S is antisymmetric. [10 points]

7. Construction Paper! [15 points]

Convert the following NFA into a DFA using the algorithm from lecture.



8. Modern DFAs [15 points]

Let $\Sigma = \{0, 1, 2\}$. Construct a DFA that recognizes exactly strings with a 0 in all positions i where $i\%3 = 0$.