Homework 3: Predicate Logic

VERSION 2: We corrected typos in lines 2, 5.2, and 5.3 of problem 7 (all the t's in the problem should have been p. This was not one of the two errors intended in the problem.

Due date: Wednesday January 26th at 10 PM
If you work with others (and you should!), remember to follow the collaboration policy outlined in the syllabus.
In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.
We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting. You are allowed to have longer explanations, but explanations significantly longer than necessary may receive deductions.

Be sure to read the grading guidelines on the assignments page for more information on what we're looking for.

This assignment has 5 pages, be sure you keep scrolling!

1. Inside Baseball

In the beforetimes, you went to a UW baseball game with two friends on "Bark at the Park" day. Husky Baseball Stadium rules do not allow for non-human mammals to attend, except as follows: (1) Dubs is allowed at every game (2) if it is "Bark at the Park" day, everyone can bring their pet dogs.

You let your domain of discourse be all mammals at the game.

The predicates Dog, Dubs, Human are true if and only if the input is a dog, Dubs, or a human respectively. UW is facing the Oregon State Beavers. The predicate $\texttt{HuskyFan}(x)$ means "x is a Husky fan" and similarly for BeaverFan. Finally HavingFun is true if and only if the input mammal is having fun right now.

1.1. Strike One [12 points]

Translate the following observations into English. Your translations should take advantage of "restricting the domain" to make more natural translations when possible, but you should not otherwise simplify the expression before translating.

(a) $\exists x\,(\texttt{HuskyFan}(x) \wedge \texttt{Human}(x) \wedge \neg\,\texttt{HavingFun}(x))$

(b) $\forall x(\texttt{BeaverFan}(x) \rightarrow \neg\,\texttt{HavingFun}(x)) \wedge \forall x(\texttt{HuskyFan}(x) \vee \texttt{Dubs}(x) \rightarrow \texttt{HavingFun}(x))$

(c) $\neg\exists x\,(\texttt{Dog}(x) \wedge \texttt{HavingFun}(x) \wedge \texttt{BeaverFan}(x))$

1.2. Strike Two [4 points]

You realize that the first sentence is false. State the negation of (a) in English. You should simplify the negation so that the English sentence is natural.

2. Become a Domain Expert [10 points]

For the following statements, translate them into predicate logic (specifying and defining any predicates use). Then provide a domain of discourse where the statement is true and another domain of discourse where the statement is false. No explanation for the domains is required, but an explanation may be provided if you think it won't be clear to us why the statements evaluate to true and false.

(a) All x who attend 311 lecture have received a PhD.

(b) For every y who visits office hours, there is a z who will answer y's question.

3. Nested Quantifiers [15 points]

Fix your domain of discourse to be "all widgets"[1] (i.e. a single element of the domain is "a widget"). There are three types of widgets: red, blue, and yellow (every widget is exactly one of those types). The predicates $\texttt{red}(x), \texttt{blue}(x), \texttt{yellow}(x)$ return true if and only if the widget is of the named type. You can also use the predicates $\texttt{free}(x), \texttt{expensive}(x), \texttt{fancy}(x), \texttt{complicated}(x)$ to say a widget is free, expensive, fancy, or complicated respectively. Finally, use $\texttt{similar}(x, y)$ to say x and y are similar.

In this problem, an example of something you might give for a "scenario" might be "all fancy widgets are blue, but not all blue widgets are fancy"

(a) Your friend tried to translate "Every red widget is expensive or fancy" and got

$$\forall x(\texttt{red}(x) \wedge \texttt{expensive}(x) \wedge \texttt{fancy}(x)).$$

The translation is incorrect. Give a correct translation, and describe a scenario (i.e. facts about widgets) in which your translation and their translation evaluate to different truth values.

(b) Your friend tried to translate "There is a blue widget that is similar to all yellow widgets" and got

$$\exists x \forall y([\texttt{blue}(x) \wedge \texttt{yellow}(y)] \rightarrow \texttt{similar}(x, y)).$$

The translation is incorrect. Give a correct translation, and describe a scenario (i.e. facts about widgets) in which your translation and their translation evaluate to different truth values.

(c) Translate the sentence "For every blue widget, there is a fancy widget such that for all red widgets: the blue widget and the fancy widget are similar, the red widget and blue widget are not similar, and the fancy widget is expensive" into predicate logic.

4. There is an implication [8 points]

Implications are uncommon under existential quantifiers. Consider this expression (which we'll call "the original expression"): $\exists x(\mathsf{P}(x) \rightarrow \mathsf{Q}(x))$

(a) Suppose that $\mathsf{P}(x)$ is not always true (i.e. there is an element in the domain for which $\mathsf{P}(x)$ is false). Explain why the original expression is true in this case. (1-2 sentences should suffice. If you prefer, you may give a formal proof instead).

(b) Suppose that $\mathsf{P}(x)$ is always true (i.e. $\forall x\, \mathsf{P}(x)$). There is a simpler statement which conveys the meaning of the original expression (i.e. is equivalent to it for all domains and predicates. By simpler, we mean "uses fewer symbols"). Give that expression, and briefly (1-2 sentences) explain why it works.

(c) Ponder, based on the last two parts, why it's very uncommon to write the original expression. You do not have to write anything for this part, simply ponder. [0 points]

[1] "widget" is an old-timey word for "something a machine would make" or "product." It's still occasionally used in CS as a more fancy sounding version of "thing."

5. For every iteration [6 points]

Imagine you have the predicate `pred`(x, y), which is true if and only if the java method `public boolean pred(Element x, Element y)` returns true. Write a java method that takes in a `Domain` object (which is a `list` of all the `Elements` in the domain) and returns the value of $\exists x \forall y$ `pred`(x, y)

You do not need to follow 142/143's style rules for code, but if your code is extremely unnecessarily convoluted you may lose points. We won't grade your code for java details (e.g. if you forget a semicolon, but it's clear what you meant we won't deduct; but errors that affect our understanding [say forgetting braces] may lead to deductions). You may want to consult Section 3's handout for examples of this type of code. If you're working in LaTeX you may want to use the verbatim environment (or just code in a text editor and insert a picture).

6. Spoof [14 points]

Theorem: Given $p \land \neg q$, $r \to s$, and $p \to \neg(\neg q \land s)$ prove $\neg r$.

"Spoof":

1.	$p \land \neg q$	Given
2.	p	$\land$ Elim: 1
3.	$p \to \neg(\neg q \land s)$	Given
4.	$\neg(\neg q \land s)$	Modus Ponens: 2,3
5.	$\neg\neg q \land \neg s$	DeMorgan's Law: 4
6.	$q \land \neg s$	Double Negation: 5
7.	$\neg s$	$\land$ Elim: 6
8.	$r \to s$	Given
9.	$\neg s \to \neg r$	Contrapositive: 8
10.	$\neg r$	MP: 7,9

(a) What is the most significant error in this proof? Give the line and briefly explain why it is wrong. [5 points]

(b) Show the theorem is true by fixing the error in the spoof. For this problem, please entirely rewrite the proof in your submission. [9 points]

7. Inference Proof [20 points]

Theorem: Given $s \rightarrow (p \wedge q), \neg s \rightarrow r$, and $(r \vee p) \rightarrow q$, prove q.

"Spoof:"

1.	$\neg s \rightarrow r$		Given
2.	$(r \vee p) \rightarrow q$		Given
3.	$r \rightarrow q$		Elim of $\vee$: 2
	4.1. $\neg s$	Assumption	
	4.2. r	MP: 4.1, 1	
	4.3. q	MP: 4.2, 3	
4.	$\neg s \rightarrow q$		Direct Proof Rule
	5.1. s	Assumption	
	5.2. $s \rightarrow (p \wedge q)$	Given	
	5.3. $p \wedge q$	MP: 5.1, 5.2	
	5.4. q	$\wedge$ Elim: 5.3	
5.	$s \rightarrow q$		Direct Proof Rule
6.	$(s \rightarrow q) \wedge (\neg s \rightarrow q)$		Intro $\wedge$: 5, 4
7.	$(\neg s \vee q) \wedge (\neg\neg s \vee q)$		Law of Implication
8.	$(\neg s \vee q) \wedge (s \vee q)$		Double Negation
9.	$((\neg s \vee q) \wedge s) \vee ((\neg s \vee q) \wedge q)$		Distributivity
10.	$((\neg s \vee q) \wedge s) \vee (q \wedge (\neg s \vee q))$		Commutativity
11.	$((\neg s \vee q) \wedge s) \vee (q \wedge (q \vee \neg s))$		Commutativity
12.	$((\neg s \vee q) \wedge s) \vee q$		Absorption
13.	$(s \wedge (\neg s \vee q)) \vee q$		Commutativity
14.	$((s \wedge \neg s) \vee q) \vee q$		Associativity
15.	$(\mathsf{F} \vee q) \vee q$		Negation
16.	$(q \vee \mathsf{F}) \vee q$		Commutativity
17.	$q \vee q$		Identity
18.	q		Idempotence

(a) There are two major errors in this proof. Indicate which lines contain the errors and, for each one, explain (as briefly as possible) why that line is incorrect. [8 points]

(b) Is the conclusion of the "spoof" correct (that is, is the "Theorem" true)? If it is incorrect, describe propositions p, q, r, s such that the givens are true, but the claim is false. If the conclusion is correct, briefly explain how to correct any errors in lines 1–5 (you'll explain errors in 6–18 in part c). [4 points]

(c) Give a correct proof of what is claimed in lines 6–18, i.e., that from $(s \rightarrow q) \wedge (\neg s \rightarrow q)$, we can infer that q is true. [8 points]

8. Feedback [2 points]

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you. We'll give full credit on this question for filling out the first two questions.

- How many hours did you spend working on this assignment (excluding any extra credit questions, if applicable)? Report your estimate to the nearest hour.
- Which problem did you spend the most time on?
- Any other feedback for us?