Homework 1: Propositional Logic

Due date: Wednesday January 12th at 10 PM

If you work with others (and you should!), remember to follow the collaboration policy outlined in the syllabus. In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.

We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting. You are allowed to have longer explanations, but explanations significantly longer than necessary may receive deductions.

Be sure to read the grading guidelines on the assignments page for more information on what we're looking for.

1. Syllabus [8 points]

Read the syllabus and/or watch "Lecture 0" on panopto. If you have any questions about the syllabus, ask us on Ed!

On your homework submission write "I have watched lecture 0 and/or read the syllabus." for full credit.

2. Translation [16 points]

Translate the English statements into symbolic logic, making the atomic propositions as basic as possible.

- (a) To watch *The Emperor's New Groove*, it is necessary to have a DisneyPlus account or a DVD. [4 points] *Hint: remember propositions need to unambiguously assert something. You'll need a subject and verb in all the propositions here.*
- (b) The code can be compiled only if it has no errors and the JDK is installed. [4 points]
- (c) Define a set of *at most three* atomic propositions. Then, use them to translate all of these sentences about tweedle beetle bottle battles into logic. [8 points]
 - (i) If it is called a tweedle beetle bottle battle, then the tweedle beetles battle with bottles
 - (ii) Unless they battle in a bottle, it is not called a tweedle beetle bottle battle.
 - (iii) It is called a tweedle beetle bottle battle only if they battle in a bottle or battle with bottles.

3. Trickier translation [5 points]

The following sentence is idiomatic in English...but not very clear logically. Convert the statement into propositional logic, then write an English sentence that has the same meaning, but is clearer logicially.

(a) I will buy an NFT if Robbie creates an NFT and if I sell enough Bitcoin.

4. Inequivalence [16 points]

For each part, find a truth assignment (i.e. an assignment of True or False to p, q, and r) to show the pair of statements are not equivalent. Explain why your assignments work (our explanations are 1-2 sentences).

- (a) $(q \rightarrow p) \rightarrow r \text{ vs. } q \rightarrow (r \rightarrow p).$
- (b) $p \lor (p \land q)$ vs. $p \lor q$.

(c) $(p \land q) \rightarrow r$ vs. $(\neg p \land \neg q) \lor r$

Compound Proposition [7 points] 5.

Find a compound proposition involving the variables p, q, r, and s that is true precisely when at least two of p,q,r, and s are true. Explain why your answer works (1-2 sentences). Note: By "precisely," we mean also that the proposition should be false whenever the condition is not met.

6. Proof [24 points]

In Lecture 3 (and in Lecture 2 slide 43) we gave a symbolic proof that $(p \land q) \lor (\neg p \land q) \lor (\neg p \land \neg q) \equiv (\neg p \lor q)$. In this problem we will give another proof.

- (a) Our intuition for the proof in class was "the last two pieces of the formula correspond to vacuous truth." Identify a commonality in the first two pieces of the formula, and describe it. (Your description should be similar in spirit to the one from class, but you don't need to use fancy vocabulary like "vacuous truth" – our answer here is one sentence) [4 points]
- (b) Give another proof of the formula that matches the intuition from part a instead of the intuition from class. [16 points]

Read the symbolic proof guidelines on the assignments page before you start.

Hint: your proof, if it matches your intuition from (a) will be different from the one from class – at least some of the intermediate expressions will be different.

(c) In class we labeled portions of the proof in purple with high-level descriptions of what they are doing (lecture 2 slide 43, left side). Produce similar labels for your proof. Submit your answer in the form "Steps [X] to [Y]: [label]" for each part. [4 points]

Note: The goal here is to give intuition for what is happening at a higher level than individual steps.

Highly Illogical [12 points] 7.

This problem is inspired by the Star Trek universe. A "transporter" is a teleportation machine. Food is "replicated" instead of taken out of the freezer. "Ice cream" is still delicious.

As a result of a tragic transporter accident, you have been named the legal guardian of T'Vin, a young Vulcan child. T'Vin is ruthlessly and perfectly logical – he takes each statement at its logical meaning; he does not believe you will lie, but he does not accept unstated intentions.

You want T'Vin to clean his room, and (like many parents) you've resorted to bribery - you are willing to replicate some ice cream in exchange for compliance. T'Vin loves ice cream and hates cleaning. He would rather have ice cream and clean his room than do neither, but will not clean his room without the reward of ice cream, nor will he clean if he has any hope of getting ice cream some other way.

- (a) You tell T'Vin "If you don't clean your room, then I won't replicate you any ice cream." He looks at you and says "It is not yet logical to clean my room." Why will he not comply? (1-2 sentences)
- (b) You try again: "Forget the first promise" you say "If you clean your room, then I will replicate you some ice cream." Still he says, "It is not yet logical to clean my room" Why is he not cleaning? (1-2 sentences)
- (c) Give a logical sentence (or sentences) which will compel T'Vin to clean his room. Argue that T'Vin will finally clean his room. (2-4 sentences)