1. **Warm-Up**

Translate the English sentences below into symbolic logic.

(a) If I am lifting weights this afternoon, then I do a warm-up exercise.

*Solution:*

Since we’re in “if...then...” form, the sentence is an implication.

\[ p \rightarrow q \]

\(p\): I am lifting weights

\(q\): I do a warm-up exercise

(b) If I am cold and going to bed or I am two-years old, then I carry a blanket.  

*Solution:*

\[ (p \land q) \lor r \rightarrow s \]

\(p\): I am cold

\(q\): I am going to bed

\(r\): I am two-years old

\(s\): I carry a blanket

How did we know the translation wasn’t \([p \land (q \lor r)] \rightarrow s\) ? Two hints were available: first, omitted words (“going to bed” instead of “I am going to bed” indicates \(q\) should be closer to the “and” than the “or”), second the interpretation of the sentence – two-year olds more commonly carry blankets during the day than warm adults.

2. **If I can translate, then...**

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

(a) whenever I walk my dog, I make new friends.  

*Solution:*

\(p\): I walk my dog

\(q\): I make new friends

\[ p \rightarrow q \]

The promise is that we will definitely make new friends on the condition of walking our dog.

(b) I will drink coffee, if Starbucks is open or my coffeemaker works.

*Solution:*

\(p\): I will drink coffee

\(q\): Starbucks is open

\(r\): my coffeemaker works
\((q \lor r) \rightarrow p\)

(c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote. **Solution:**

\[p: \text{One is a U.S. citizen} \quad q: \text{One is over 18} \quad r: \text{One is eligible to vote}\]

\[(p \land q) \rightarrow r\]

The original sentence omits a subject. We introduced a dummy subject “one” to the propositions, you might have said “someone” or “a person” instead (among other options).

(d) I can go home only if I have finished my homework. **Solution:**

\[p: \text{I can go home} \quad q: \text{I have finished my homework.}\]

\[p \rightarrow q\]

The promise here is that if I can go home then I must have finished my homework. It can sometimes help to imagine when the sentence is broken. Is it broken if my homework is finished, but I cannot go home? No, perhaps I also have to say bye to my friends before I leave. But if I can go home with unfinished homework, then the promise is broken.

“Only if” is one of the more confusing arrangements – the consequence (“the then part”) is adjacent to the “only if.”

(e) Having an internet connection is necessary to log onto zoom. **Solution:**

\[p: \text{One has an internet connection} \quad q: \text{One can log onto zoom}\]

\[q \rightarrow p\]

The internet connection is not enough (what if you don’t have the meeting link?) but certainly if you are in the meeting then you have a connection.

(f) I am a student because I attend university. **Solution:**

\[p: \text{I am a student} \quad q: \text{I attend university}\]

\[q \rightarrow p\]

This can be understood that since I attend university, then necessarily I am a student. You could also be a student if you do not attend a university, for example if you attend high school, but necessarily if you attend university then you must be a student.

3. **I can rewrite these formulas in English, only if...**

Given propositions and a logical formula, write two potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.
(a) \( p \): The sun is out \\ \qquad q: \text{We have class outside} \\ \quad \quad p \rightarrow q \\

Solution:

If the sun is out, then we have class outside. Whenever the sun is out, we have class outside.

(b) \( p \): the book has been out for a week. \\ \qquad q: I don’t have homework. \\ \qquad r: I have finished reading the book. \\ \quad \quad (p \land q) \rightarrow r \\

Solution:

I have finished reading the book, if it has been out for a week and I don’t have homework. The book being out for a week and me not having homework is sufficient for me to have finished reading the book.

(c) \( p \): I have read the manual \\ \qquad q: I operate the machine \\ \quad \quad q \rightarrow p \\

Solution:

I operate the machine only if I have read the manual. Operating the machine implies that I have read the manual.

4. Translation

For each of the following, define propositional variables and translate the sentences into logical notation.

(a) I will remember to send you the address only if you send me an e-mail message. Solution:

\[
p: \text{I will remember to send you the address} \\
q: \text{You send me an e-mail message} \\
\quad p \rightarrow q
\]

(b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area. Solution:

\[
p: \text{Berries are ripe along the trail} \\
q: \text{Hiking is safe} \\
r: \text{Grizzly bears have not been seen in the area}
\]
(c) Unless I am trying to type something, my cat is either eating or sleeping. **Solution:**

\[
p \rightarrow (q \leftrightarrow r)
\]

\[
\neg r \rightarrow (p \oplus q)
\]

5. **Tea Time**

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

(a) Define propositional variables and translate the sentence into an expression in logical notation. **Solution:**

\[
p : I \text{ am drinking tea} \\
q : I \text{ am eating a cookie}
\]

\[
(p \rightarrow q) \lor (q \rightarrow p)
\]

(b) Fill out a truth table for your expression. **Solution:**

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6. **Exclusive Or**

Exclusive or (⊕) and inclusive or (∨) both can be translated as “or” in English. For each of the following ambiguous phrases, decide which type of “or” is likely meant and why.

(a) Experience with C or Java is required. **Solution:**

Inclusive or. Experience with both is usually not a bad thing.
(b) Lunch includes soup or salad.  **Solution:**

Exclusive or. Most restaurants charge you more for both.

(c) Publish or perish.  **Solution:**

This phrase is a common one among researchers – it means publish papers or your career will perish. Exclusive or is meant; i.e. if you do indeed publish you should avoid the loss of your career.

(d) To enter the country, you need a passport or voter registration card.  **Solution:**

Inclusive or – if you have both, they won’t kick you out!