Full outline

1. Suppose for the sake of contradiction that $L$ is regular. Then there is some DFA $M$ that recognizes $L$.

2. Let $S$ be [fill in with an infinite set of prefixes].

3. Because the DFA is finite and $S$ is infinite, there are two (different) strings $x, y$ in $S$ such that $x$ and $y$ go to the same state when read by $M$ [you don’t get to control $x, y$ other than having them not equal and in $S$]

4. Consider the string $z$ [argue exactly one of $xz, yz$ will be in $L$]

5. Since $x, y$ both end up in the same state, and we appended the same $z$, both $xz$ and $yz$ end up in the same state of $M$. Since $xz \in L$ and $yz \notin L$, $M$ does not recognize $L$. But that’s a contradiction!

6. So $L$ must be an irregular language.

Let’s Try another

The set of strings with balanced parentheses is not regular.

What do you want $S$ to be? What would you have to count?

The number of unclosed parentheses.
Want $S$ to be a set with infinitely many strings with different numbers of unclosed parentheses.
Let $S = (^{*}$