Stamp Collection, Done Wrong

Define $P(n)$ I can make n cents of stamps with just 4 and 5 cent stamps.

We prove $P(n)$ is true for all $n \geq 12$ by induction on n.

Base Case:

12 cents can be made with three 4 cent stamps.
Inductive Hypothesis Suppose $P(k)$, $k \geq 12$.

Inductive Step:

We want to make $k + 1$ cents of stamps. By IH we can make k cents exactly with stamps. Replace one of the 4 cent stamps with a 5 cent stamp.

$P(n)$ holds for all n by the principle of induction.

Fibonacci Inequality

Show that $f(n) \leq 2^n$ for all $n \geq 0$ by induction.

$$f(0) = 1; \qquad f(1) = 1$$
$$f(n) = f(n-1) + f(n-2) \text{ for all } n \in \mathbb{N}, n \geq 2.$$

Claim: $3|(2^{2n}-1)$ for all $n \in \mathbb{N}$.

[Define $P(n)$]

Base Case
Inductive Hypothesis
Inductive Step

[conclusion]

Induction: Hats!

You have n people in a line ($n \geq 2$). Each of them wears either a purple hat or a gold hat. The person at the front of the line wears a purple hat. The person at the back of the line wears a gold hat.

Show that for every arrangement of the line satisfying the rule above, there is a person with a purple hat next to someone with a gold hat.

Yes this is kinda obvious. I promise this is good induction practice.

Yes you could argue this by contradiction. I promise this is good induction practice.