How do we know recursion works?

//Assume i is a nonnegative integer
//returns 2^i.
public int CalculatesTwoToTheI(int i){
    if(i == 0)
        return 1;
    else
        return 2*CalculatesTwoToTheI(i-1);
}

Why does CalculatesTwoToTheI(4) calculate 2^4?
Convince the people around you!

Making Induction Proofs Pretty

All of our induction proofs will come in 5 easy(?) steps!
1. Define \( P(n) \). State that your proof is by induction on \( n \).
2. Show \( P(0) \) i.e. show the base case
3. Suppose \( P(k) \) for an arbitrary \( k \).
4. Show \( P(k + 1) \) (i.e. get \( P(k) \to P(k + 1) \))
5. Conclude by saying \( P(n) \) is true for all \( n \) by induction.
The Principle of Induction (formally)

\[ P(0); \forall k (P(k) \rightarrow P(k + 1)) \]
\[ \therefore \forall n (P(n)) \]

Informally: if you knock over one domino, and every domino knocks over the next one, then all your dominoes fell over.

More Induction

Induction doesn’t only work for code!

Show that \[ \sum_{i=0}^{n} 2^i = 1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1. \]