

You Try!

Claim: for all $a, b, c, n \in \mathbb{Z}, n > 0$: If $a \equiv b \pmod{n}$ then $ac \equiv bc \pmod{n}$

Before we start we must know:

1. What every word in the statement means.
2. What the statement as a whole means.
3. Where to start.
4. What your target is.

Divides

For integers x, y we say $x|y$ (" x divides y ") iff there is an integer z such that $xz = y$.

Equivalence in modular arithmetic

Let $a \in \mathbb{Z}, b \in \mathbb{Z}, n \in \mathbb{Z}$ and $n > 0$.
We say $a \equiv b \pmod{n}$ if and only if $n|(b - a)$

Warm-up

Show that if $a \equiv b \pmod{n}$ then $b \equiv a \pmod{n}$.

Now that we've proven this, we aren't going to care whether you write $n|(b - a)$ or $n|(a - b)$ when you write the definition.

We can't remember the right order either.

Another Proof

For all integers, a, b, c : Show that if $a \nmid (bc)$ then $a \nmid b$ or $a \nmid c$.

Proof:

Let a, b, c be arbitrary integers, and suppose $a \nmid (bc)$.

Then there is not an integer z such that $az = bc$

...

So $a \nmid b$ or $a \nmid c$

A bad proof

Claim: if x is positive then $x + 5 = -x - 5$.

$$x + 5 = -x - 5$$

$$|x + 5| = |-x - 5|$$

$$|x + 5| = |-(x + 5)|$$

$$|x + 5| = |x + 5|$$

$$0 = 0$$

This claim is **false** – if you're trying to do algebra, you need to start with an equation you know (say $x = x$ or $2 = 2$ or $0 = 0$) and expand to the equation you want.