Try it yourselves

Every cat loves some human. There is a cat that loves every human.

Let your domain of discourse be mammals. Use the predicates $\text{Cat}(x), \text{Dog}(x)$, and $\text{Loves}(x, y)$ to mean $x$ loves $y$.

Try it!

Given: $p \lor q, (r \land s) \rightarrow \neg q, r$. Show: $s \rightarrow p$

![Proof Diagram]

You can still use all the propositional logic equivalences too!
Proof Using Quantifiers

Suppose we know $\exists x P(x)$ and $\forall y [ P(y) \rightarrow Q(y) ]$. Conclude $\exists x Q(x)$.

\[
\begin{array}{c}
\text{Intro } \exists \\
P(c) \text{ for some } c \\
\hline
\therefore \exists x P(x) \\
\exists x P(x)
\end{array}
\]

\[
\begin{array}{c}
\text{Eliminate } \exists \\
\forall x P(x) \\
\hline
\therefore P(c) \text{ for a fresh } c \\
\forall x P(x)
\end{array}
\]

\[
\begin{array}{c}
\text{Eliminate } \forall \\
\forall x P(x) \\
\hline
\therefore P(a) \text{ for any } a \\
\forall x P(x)
\end{array}
\]

\[
\begin{array}{c}
\text{Intro } \forall \\
P(a); a \text{ is arbitrary} \\
\hline
\therefore \forall x P(x)
\end{array}
\]

Find The Bug

Let your domain of discourse be integers.
We claim that given $\forall x \exists y \text{ Greater}(y, x)$, we can conclude $\exists y \forall x \text{ Greater}(y, x)$
Where $\text{Greater}(y, x)$ means $y > x$

1. $\forall x \exists y \text{ Greater}(y, x)$ Given
2. Let $a$ be an arbitrary integer --
3. $\exists y \text{ Greater}(y, a)$ Elim $\forall$ (1)
4. $b \geq a$ Elim $\exists$ (2)
5. $\forall x \text{ Greater}(b, x)$ Intro $\forall$ (4)
6. $\exists y \forall x \text{ Greater}(y, x)$ Intro $\exists$ (5)