Contrapositive

We showed $p \to q \equiv \neg q \to \neg p$ with a truth table. Let's do a proof.

Try this one on your own. Remember

1. Know what you're trying to show.
2. Stay on target – take steps to get closer to your goal.

Hint: think about your tools.

There are lots of rules with AND/OR/NOT,

but very few with implications…

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Help me adjust my explanation!

Properties of Logical Connectives

For every propositions p, q, r the following hold:

- **Identity**
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- **Domination**
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$
- **DeMorgan's Laws**
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- **Double Negation**

  $\neg\neg p \equiv p$
- **Law of Implication**

  $p \to q \equiv \neg p \vee q$
- **Contrapositive**

  $p \to q \equiv \neg q \to \neg p$

Try it…

What's a possible domain of discourse for these lists of predicates?

1. "x is a cat", "x barks", "x likes to take walks"
2. "x is prime", "x=5" "$x < 20$" "x is a power of two"
3. "x is enrolled in course y", "y is a pre-req for z"

Translations

"For every x, if x is even, then $x = 2$."

"There are x, y such that $\mathrm{x} < y$."

$\exists x\ (\texttt{Odd}(x) \wedge \texttt{LessThan}(x, 5))$

$\forall y\ (\mathrm{Even}(y) \wedge \mathrm{Odd}(y))$

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Help me adjust my explanation!