

# Section 10: FSMs, Irregularity, and Cardinality

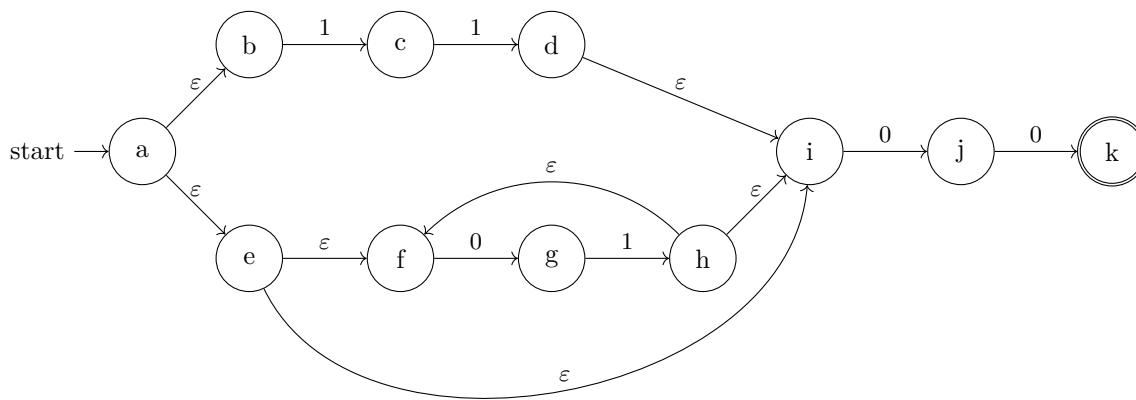
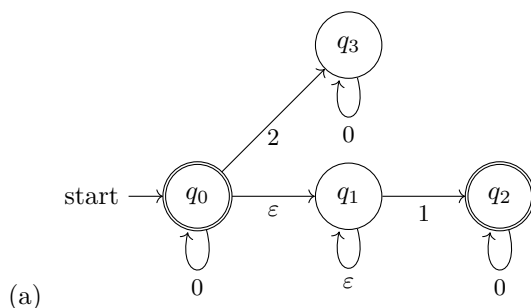
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## 1. RE to NFA

Convert the regular expression “ $(11 \cup (01)^*)00$ ” to an NFA using the algorithm from lecture.

## 2. NFAs to DFAs

Convert each of the following NFAs to DFAs.



(b)

## 3. Irregularity

(a) Let  $\Sigma = \{0, 1\}$ . Prove that  $\{0^n 1^n 0^n : n \geq 0\}$  is not regular.

(b) Let  $\Sigma = \{0, 1, 2\}$ . Prove that  $\{0^n (12)^m : n \geq m \geq 0\}$  is not regular.

## 4. Cardinality

(a) You are a pirate. You begin in a square on a 2D grid which is infinite in all directions. In other words, wherever you are, you may move up, down, left, or right. Some single square on the infinite grid has treasure on it. Find a way to ensure you find the treasure in finitely many moves.

(b) Prove that  $\{3x : x \in \mathbb{N}\}$  is countable.

(c) Prove that the set of irrational numbers is uncountable.

**Hint:** Use the fact that the rationals are countable and that the reals are uncountable.

(d) Prove that  $\mathcal{P}(\mathbb{N})$  is uncountable.

## 5. Countable Unions

(a) Show that  $\mathbb{N} \times \mathbb{N}$  is countable.

Hint: How did we show the rationals were countable?

(b) Show that the countable union of countable sets is countable. That is, given a collection of sets  $S_1, S_2, S_3, \dots$  such that  $S_i$  is countable for all  $i \in \mathbb{N}$ , show that

$$S = S_1 \cup S_2 \cup \dots = \{x : x \in S_i \text{ for some } i\}$$

is countable.

Hint: Find a way labeling the elements and see if you can apply the previous part to construct an onto function from  $\mathbb{N}$  to  $S$ .

## 6. Review: Number Theory

Let the domain of discourse be positive integers and let  $z = \gcd(m, n)$ . Consider the following claim:

$$\forall n \forall m \forall a \forall b \forall c ((a \equiv_m b \wedge a \equiv_n c) \rightarrow (b \equiv_z c))$$

(a) Translate the claim into English.

(b) Write a formal proof that the claim holds. You may use the following fact:

$$\mathbf{Fact:} \quad (\gcd(a, b) \mid a) \wedge (\gcd(a, b) \mid b)$$

(c) Translate your proof to English.