Section 09 (Part B): CFGs, Relations and FSMs

1. Reflexivity Proof

Let R and S be relations on a set A. Consider the following claim:

Given that R and S are reflexive, it follows that $R \cup S$ is reflexive.

(a) What does "$R \cup S$ is reflexive" become if we unroll the definition of reflexive?

(b) Write a formal proof that the claim from (a) holds.

(c) Translate the formal proof into an English proof of the original claim.

2. Relations

Suppose that R is reflexive. Prove that $R \subseteq R^2$, with a formal proof and then an English proof. You may use the following definition:

$$R^2 := R \circ R$$

3. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1, 2, 3\}$.

(a) All binary strings.

(b) All strings whose digits sum to an even number.

(c) All strings whose digits sum to an odd number.

4. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

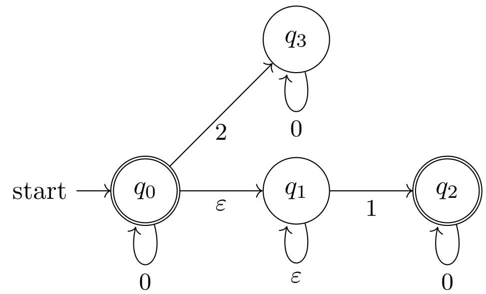
(a) All strings which do not contain the substring 101.

(b) All strings containing at least two 0's and at most one 1.

(c) All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

5. NFAs

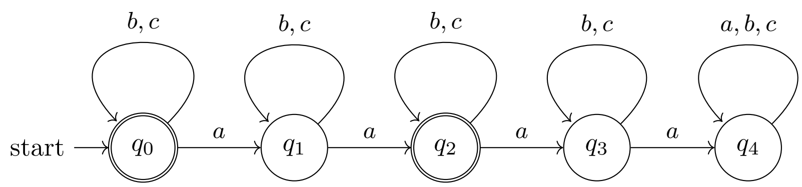
(a) What language does the following NFA accept?



(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

6. DFA Minimization

(a) Minimize the following DFA:



(b) Minimize your solution to problem 1.